



A LEVEL MATHEMATICS

Summer Task 2023

So you're thinking about choosing Mathematics as an A level. This task has been put together to help give you the best possible start in September by hitting the ground running.

This task concentrates on material that you should have seen at GCSE. There are some topics such as trigonometry that do not feature in this task, but you will be required to know them for the course.

It is expected that you will complete ALL the questions/problems in this task and submit your work during your first mathematics lesson in September. It is expected that this will be done on separate paper with full working shown. This will form part of your initial assessment grade so it is very important that it is done to the best of your ability. You can email maths@springwoodhighschool.co.uk any questions relating to the task or the A-level course.

We would like to wish you all the best for your results in August and for your future studies.

Best Wishes,

Springwood High School Mathematics Department

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SECTION 1 – NUMBER

1. $\frac{3}{4} + \frac{1}{3}$
2. $5\frac{1}{3} - \frac{3}{2}$
3. $2\frac{2}{3} \times \frac{1}{4}$
4. $5\frac{1}{3} \div 2\frac{1}{4}$
5. Simplify: (a) $\sqrt{18}$ (b) $\sqrt{48}$
6. Simplify: (a) $\sqrt{6} \times \sqrt{15}$ (b) $\sqrt{10} \times \sqrt{15}$
7. Simplify: (a) $\sqrt{27} + \sqrt{12}$ (b) $\sqrt{125} - \sqrt{75}$
8. Simplify: (a) $\frac{10}{\sqrt{5}}$ (b) $\frac{\sqrt{5}}{\sqrt{3}}$ (c) $\frac{2\sqrt{3}+5}{\sqrt{3}}$ (d) $\frac{2\sqrt{15}}{\sqrt{12}}$ (e) $\frac{12\sqrt{3}-6\sqrt{2}}{\sqrt{6}}$
9. Expand brackets and simplify (a) $(\sqrt{5} - 2)(2\sqrt{5} - 1)$
(b) $(3\sqrt{5} - \sqrt{7})^2$ (c) $(2\sqrt{11} - 3\sqrt{6})(2\sqrt{11} + 3\sqrt{6})$
10. (a) Show that $x = 1 + \sqrt{5}$ is a solution of the quadratic equation $x^2 - 2x - 4 = 0$.
(b) A right-angled triangle has the two shorter sides $\sqrt{3} - 1$ and $\sqrt{3} + 1$. Show that the hypotenuse has length $2\sqrt{2}$.
(c) A rectangle has two sides $\sqrt{7} - 1$ and $x\sqrt{7} + 2$. Its area is 12 square units. Show that $x = 2$.
11. Express the following in their simplest form. Please include your workings.
(a) $9^{1/2}$ (b) $81^{1/4}$ (c) 27^0 (d) $4^{-3/2}$ (e) $64^{5/6}$
(f) $b^5 \times b^6$ (g) g^7/g^3 (h) $(x^2)^6$
(i) $(125/8)^{1/3}$ (j) $(16/9)^{-3/2}$

SECTION 2 – ALGEBRA

1. Expand and simplify

a. $3y(4x - 2y)$

c. $(2x - 4)(2x + 4)$

e. $(x - 2)(2x + 3)(5x + 3)$

b. $(x + 4)(x - 3)$

d. $(3x - 1)(x + 2) - 3x(2x - 4)$

2. Factorise

a. $20x^2 - 4x$

c. $y^3 + 3y^2 - 4y$

e. $4x(2x + 3) - 3(2x + 3)$

g. $15x^2 + 19x + 6$

b. $8x^2y + 28xy^2$

d. $2x^2 + 5x + 3$

f. $x^2 - 144$

h. $9y^2 - 4x^2$

3. Simplify these fractions (write as a single fraction if necessary)

a. $\frac{3x^2}{7x}$

c. $3 + \frac{2}{x}$

e. $\frac{1}{x+1} - \frac{3}{x-2}$

g. $\frac{x+3}{x^2} \times \frac{x}{4}$

b. $\frac{8x^2(x+3)}{4x}$

d. $\frac{a}{b} - \frac{2a}{3b}$

f. $\frac{x(x-3)}{3} \div \frac{x-3}{x}$

h. $12x(x+2) \div \frac{3x+6}{x}$

4. a. Make a the subject of the formula $b(a + 2) = 4$

b. Make C the subject of the formula $F = \frac{9}{5}C + 32$

c. Make z the subject of the formula $\frac{z}{z+4} = \frac{3}{y}$

d. Make x the subject of the formula $y = 3\sqrt{\frac{x}{2}}$

SECTION 3 - GRAPHS

1. Find the distance between $P(2,6)$ and $Q(5,14)$.
2. Find the lengths of the sides of the triangle PQR which has vertices at $P(-4,2)$, $Q(-1, 6)$ and $R(3,3)$. What kind of triangle is it?
3. Find the equation of the line with gradient 3 and intercept 5.
4. Find the equation of the line that has gradient -2 and goes through $(0,6)$
5. Find the gradient of the lines that pass through the following points
 - a. $(5,6)$ and $(9,15)$
 - b. $(2,12)$ and $(4,1)$
 - c. $(-4,8)$ and $(10,-3)$
6. Find the equation of the straight line that passes through the points $(2,2)$ and $(6,14)$
7. Find the equation of the straight line that passes through the points $(-2,3)$ and $(6,-4)$
8. Find the equation of the line that is parallel to $y = 2x - 6$ that goes through the point $(1,12)$
9. Find the equation of the line that is parallel to $2y = 3x - 4$ that goes through the point $(2,4)$
10. Find the equation of the line that is perpendicular to $y = 2x - 6$ that goes through the point $(1, 1)$

SECTION 4 – EQUATIONS

1. Solve the following linear equations

a. $5x + 3 = 3x + 17$ b. $3x - 11 = 3 - x$ c. $3(x + 2) + 2(2x - 5) = 5(x - 1) + 9$

2. Solve the following quadratic equations (to 2dp) by **using the quadratic formula**.

a. $x^2 - x - 5 = 0$ b. $2x^2 - 7x - 1 = 0$
c. $x^2 = 3x + 5$ d. $-3x^2 + 2x + 1 = 0$

3. Solve the following quadratic equations by **factorizing**

a. $x^2 + 3x + 2 = 0$ b. $x^2 + 4x - 12 = 0$ c. $x^2 - 4x - 12 = 0$
d. $x^2 - 14x + 40 = 0$ e. $5x^2 + 13x + 6 = 0$ f. $3x^2 - 16x + 21 = 0$

4. Solve the following quadratic equations by **completing the square**

a. $x^2 + 6x + 1 = 0$ c. $x^2 - 20x + 34 = 0$

5. Solve the following simultaneous equations

a. $5x + 3y = 17$ b. $7x - 3y = 48$ c. $x = 2y + 1$
 $4x + 10y = 25$ $2x + y = 5$ $3x - 4y = 7$

6. Solve the following inequalities

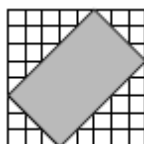
a. $3x - 8 \leq 30 + x$ b. $10 - 3x < 30 + 2x$ c. $x^2 + 12x - 28 > 0$

7. Solve the following simultaneous equations

a. $y = 4x + 7$ b. $y = x - 5$
 $y = 2x^2 + 1$ $x^2 + y^2 = 17$

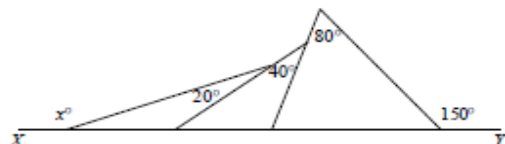
SECTION 5 – GENERAL QUESTIONS

Question 1.



The diagram shows a rectangle placed on a grid of $1\text{ cm} \times 1\text{ cm}$ squares. What is the area of the rectangle in cm^2 ?

Question 2.

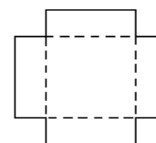


In the diagram, XY is a straight line. What is the value of x ?

Question 3.

A square piece of card has a square of side 2 cm cut out from each of its corners. The remaining card is then folded along the dotted lines shown to form an open box whose total internal surface area is 180 cm^2 .

What is the volume of the open box in cm^3 ?



Question 4.

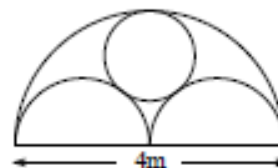
The three blind mice stole a piece of cheese. In the night, the first mouse ate $\frac{1}{3}$ of the cheese. Later, the second mouse ate $\frac{1}{3}$ of the remaining cheese. Finally, the third mouse ate $\frac{1}{3}$ of what was then left of the cheese.

Between them, what fraction of the cheese did they eat?

Question 5.

A window frame in Salt's Mill consists of two equal semicircles and a circle inside a large semicircle with each touching the other three as shown. The width of the frame is 4 m .

What is the radius of the circle, in metres?



Question 6.

You are given that $5^p = 9$, $9^q = 12$, $12^r = 16$, $16^s = 20$ and $20^t = 25$. What is the value of $pqrst$?