

Solving simultaneous equations graphically

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

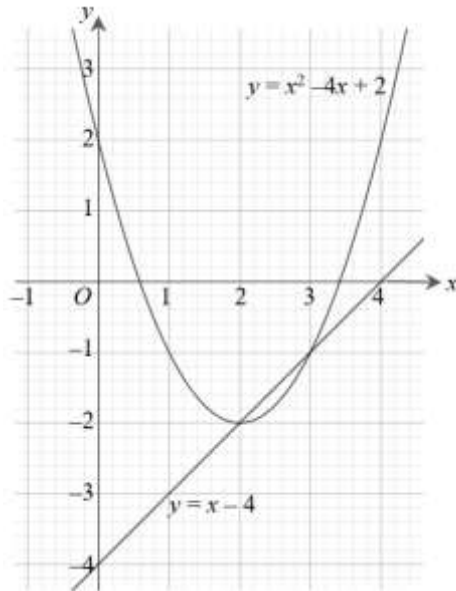
Examples

Example 1 Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

<p>$y = 5 - x$</p> <p>$y = 5 - x$ has gradient -1 and y-intercept 5. $y = 5x + 2$ has gradient 5 and y-intercept 2.</p> <p>Lines intersect at $x = 0.5, y = 4.5$</p> <p>Check: First equation $y = 5x + 2$: $4.5 = 5 \times 0.5 + 2$ YES Second equation $x + y = 5$: $0.5 + 4.5 = 5$ YES</p>	<ol style="list-style-type: none"> Rearrange the equation $x + y = 5$ to make y the subject. Plot both graphs on the same grid using the gradients and y-intercepts. The solutions of the simultaneous equations are the point of intersection. Check your solutions by substituting the values into both equations.
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Example 2 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

x	0	1	2	3	4
y	2	-1	-2	-1	2



The line and curve intersect at
 $x = 3, y = -1$ and $x = 2, y = -2$

Check:

First equation $y = x - 4$:

$$-1 = 3 - 4 \quad \text{YES}$$

$$-2 = 2 - 4 \quad \text{YES}$$

Second equation $y = x^2 - 4x + 2$:

$$-1 = 3^2 - 4 \times 3 + 2 \quad \text{YES}$$

$$-2 = 2^2 - 4 \times 2 + 2 \quad \text{YES}$$

1 Construct a table of values and calculate the points for the quadratic equation.

2 Plot the graph.

3 Plot the linear graph on the same grid using the gradient and y-intercept.
 $y = x - 4$ has gradient 1 and y-intercept -4 .

4 The solutions of the simultaneous equations are the points of intersection.

5 Check your solutions by substituting the values into both equations.

Practice

1 Solve these pairs of simultaneous equations graphically.

a $y = 3x - 1$ and $y = x + 3$

b $y = x - 5$ and $y = 7 - 5x$

c $y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

a $x + y = 0$ and $y = 2x + 6$

b $4x + 2y = 3$ and $y = 3x - 1$

c $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

- 3 Solve these pairs of simultaneous equations graphically.
- a $y = x - 1$ and $y = x^2 - 4x + 3$
 - b $y = 1 - 3x$ and $y = x^2 - 3x - 3$
 - c $y = 3 - x$ and $y = x^2 + 2x + 5$
- 4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

- 5 a Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$
- i graphically
 - ii algebraically to 2 decimal places.
- b Which method gives the more accurate solutions? Explain your answer.

Answers

- 1**
- a** $x = 2, y = 5$
 - b** $x = 2, y = -3$
 - c** $x = -0.5, y = 2.5$
- 2**
- a** $x = -2, y = 2$
 - b** $x = 0.5, y = 0.5$
 - c** $x = -1, y = -2$
- 3**
- a** $x = 1, y = 0$ and $x = 4, y = 3$
 - b** $x = -2, y = 7$ and $x = 2, y = -5$
 - c** $x = -2, y = 5$ and $x = -1, y = 4$
- 4** $x = -3, y = 4$ and $x = 4, y = -3$
- 5**
- a**
 - i** $x = 2.5, y = -2$ and $x = -0.5, y = 4$
 - ii** $x = 2.41, y = -1.83$ and $x = -0.41, y = 3.83$
 - b** Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.