MechYr2 Chapter 7 ::
Applications of Forces

jfrost@tiffin.kingston.sch.uk

www.drfrostmaths.com
@DrFrostMaths

Last modified: 3rd August 2018
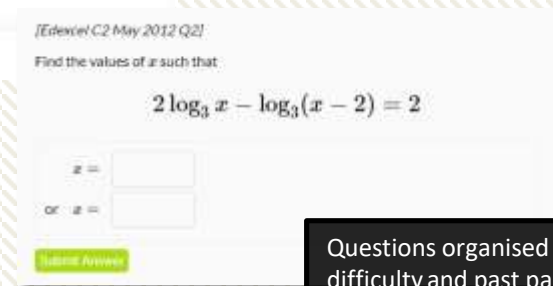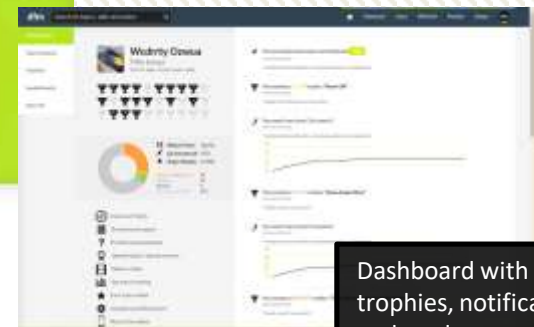
www.drfrostmaths.com

Everything is **completely free**.
Why not register?

Register now to interactively practise questions on this topic, including past paper questions and extension questions (including MAT + UKMT). Teachers: you can create student accounts (or students can register themselves), to set work, monitor progress and even create worksheets.

With questions by:

edexcel

OCR
Oxford Cambridge and RSA

AQA

UKMT

Dashboard with points, trophies, notifications and student progress.

Questions organised by topic, difficulty and past paper.

Teaching videos with topic tests to check understanding.

[Edexcel C2 May 2012 Q2]

Find the values of x such that

$$2 \log_3 x - \log_3(x-2) = 2$$

$x =$

or $x =$

Overview

There is nothing new in this chapter – it just brings together all the individual components we have learnt so far regarding forces: friction, components of forces, $F = ma$, inclined planes and connected particles, for different common types of problems.

1:: Unknown forces for bodies in equilibrium.

"If the particle is in equilibrium, determine the magnitude of the force X."
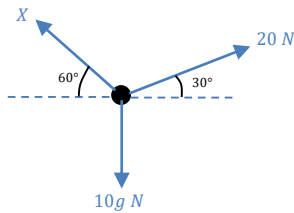
2:: Static problem involving weight, tension and pulleys

A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at $30°$ to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of $30°$ to the horizontal, as shown in Figure 3.
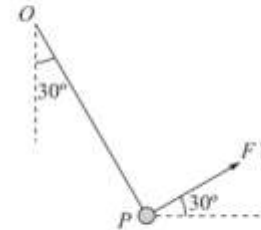
Find

(i) the value of F,

(ii) the tension in the string.

(8)

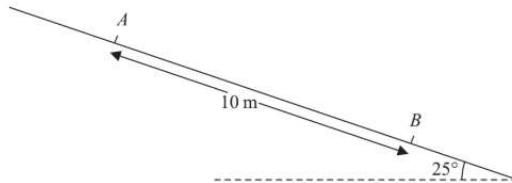**Figure 3**

3:: Objects in motion on inclined planes

Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at $25°$ to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B, (3)

(b) the acceleration of P, (2)

(c) the coefficient of friction between P and the plane. (5)
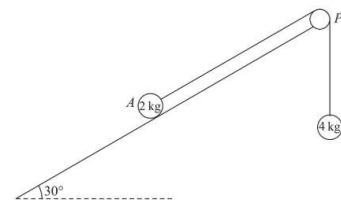
## 4:: Connected particles requiring resolution of forces.

Figure 2

A fixed rough plane is inclined at $30°$ to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

Finding unknown forces by resolving forces

The diagram shows a particle in equilibrium under the action of four forces as shown in the diagram below. The particle rests on an inclined plane which is set at an angle of 30° to the horizontal.
Find the magnitude of force F and the size of the angle, α, in degrees giving both answers to two significant figures.
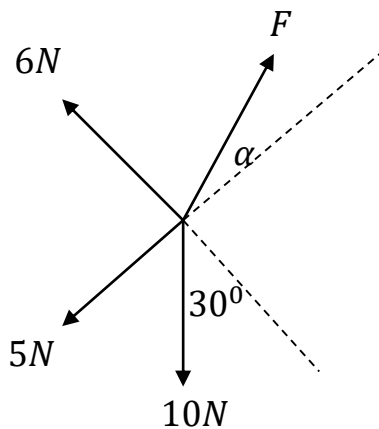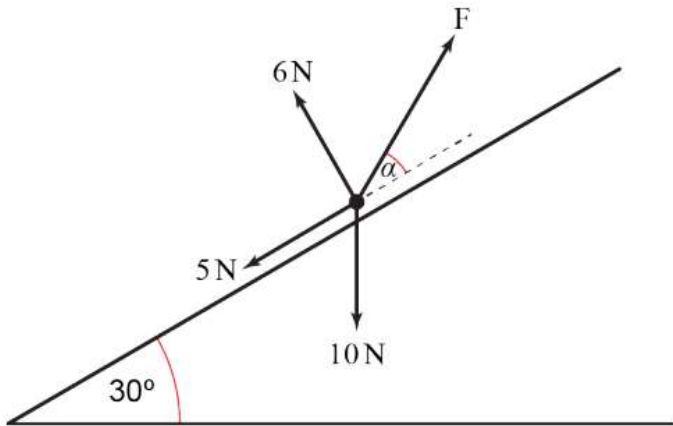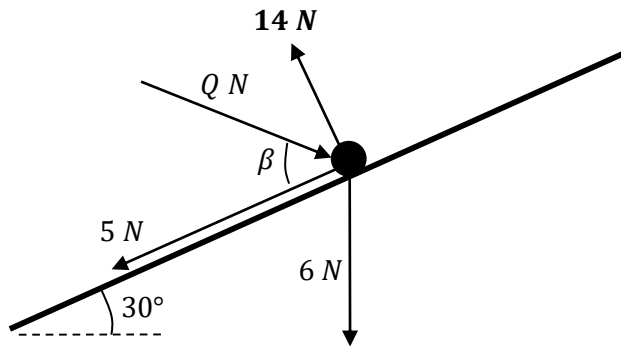

Resolve ‖ and ⊥ to the plane to set up simultaneous equations in F and α.

?

?

?

?

The diagram shows a particle in equilibrium on an inclined plane under the forces shown. Find the magnitude of the force Q and the size of the angle β.

14 N

Q N

β

5 N

6 N

30°

?

?

?

Exercise 7A

Pearson Stats/Mechanics Year 2
Pages 130-132

Modelling with Statics

We can apply this to problems involving tension, weight and pulleys.

A light, inextensible string of length 50cm has its upper end fixed at a point A and comes with a particle of mass 8kg at its lower end. A horizontal force P applied to the particle keeps it in equilibrium 30cm from the vertical through A.

By resolving horizontally and vertically, find the magnitude of P and the tension in the string.

? Force Diagram

Using Pythagoras, the missing length is   ?

$$\sin\theta = \boxed{?} \quad and \cos\theta = \boxed{?}$$

?

?

Further Example

A light, inextensible string passes over a smooth pulley fixed at the top of a smooth plane inclined at 30° to the horizontal. A particle of mass 2kg is attached to one end of the string and hangs freely. A mass m is attached to the other end of the string and rests in equilibrium on the surface of the plane. Calculate the normal reaction between the mass m and the plane, the tension in the string and the value of m.

? Force Diagram

For B:

?

For A:
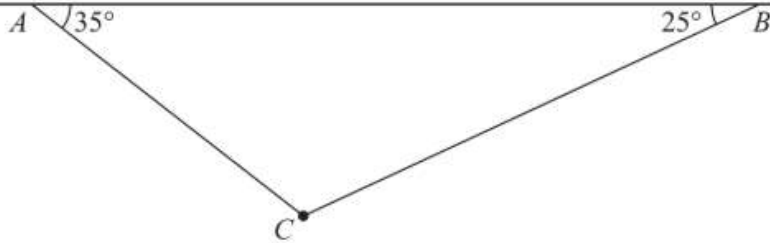
?

?

Test Your Understanding

Figure 1

A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

(i) the tension in the string AC,

(ii) the tension in the string BC.

(8)

The particle can't move along the string, so we have two separate strings with separate tensions. Introduce suitable variables for the tensions of each, e.g. T_1 and T_2.

?

Exercise 7B

Pearson Stats/Mechanics Year 2
Pages 134-137

Friction and Static Particles

Earlier in the module we saw that the frictional force $F \leq \mu R$, where $F = \mu R$ if the object on the plane is moving. Were the object is not moving, we saw that the **force of friction acts in a direction opposite** to that which it would be moving if the frictional force wasn't there.

A 10kg truck lies on a horizontal rough floor. The coefficient of friction between the trunk and the floor is $\frac{\sqrt{3}}{4}$.

Calculate the magnitude of the force, P, which is necessary to pull the trunk horizontally if P is applied:
a) horizontally
b) at 30⁰ above the horizontal

a

? Force Diagram

? Working

b

? Force Diagram

? Working

Example – Rough Inclined Plane

A mass of 6kg rests in limiting equilibrium on a rough plane inclined at 30° to the horizontal.
a) Find the coefficient of friction between the mass and the plane.
b) A horizontal force of magnitude P N is applied to the box. Given that the box remains in equilibrium, find the maximum possible value of P.

a

? Force Diagram

? Working

b

? Force Diagram

? Working

Pearson Stats/Mechanics Year 2
Pages 139-142

Static rigid bodies

Recall from the chapter on moments that for a stationary rigid body:
- The **resultant force is 0**.
- The **resultant moment is 0**.

The problems are the same as in the moments chapter, except now we may need to consider frictional forces.

A uniform rod AB of mass 45kg and length 12m rests with the end A on rough horizontal ground. The rod rests against a smooth peg C where $AC = 8$ m. The rod is in limiting equilibrium at an angle of $15°$ to the horizontal. Find:

(a) the magnitude of the reaction of C
(b) the coefficient of friction between the rod and the ground.

? Diagram
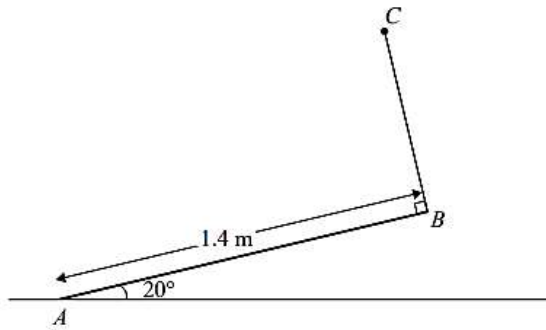
a

?

b

?

Further Example

C

1.4 m

20°

B

A

Figure 2

A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

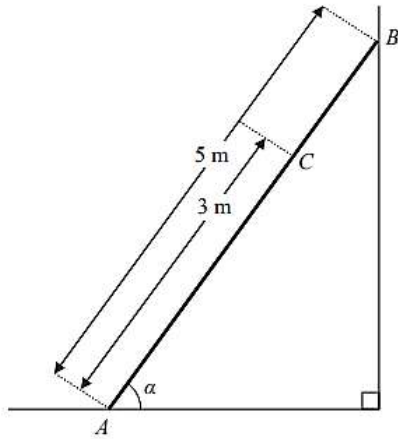(b) find the coefficient of friction between the rod and the ground.

(7)

? Diagram

b

?

a

?

A ladder, of length 5 m and mass 18 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \dfrac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point C, where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

Find the value of μ.

(9)

Figure 1

? Diagram

?

Pearson Stats/Mechanics Year 2
Pages 144-146

Particles moving on a rough plane

If a particle is in motion, $F = F_{max} = \mu R$, and F opposes the direction of motion.
- Draw a force diagram
- Use Newton's 2nd law to resolve parallel and perpendicular to the plane (sometimes it is easier to resolve vertically and horizontally)
- Use SUVAT to solve problems if F (and therefore a) are constant

A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle,

(3)

(b) the coefficient of friction between the particle and the plane.

(5)

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 3. The particle is in equilibrium and on the point of moving up the plane.
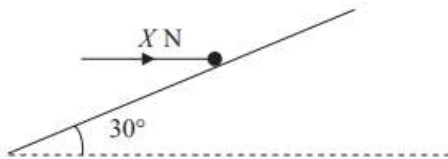

Figure 3

(c) Find the value of X.

(7)

a

?

Particles moving on a rough plane

b

A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle,

(3)

(b) the coefficient of friction between the particle and the plane.

(5)

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 3. The particle is in equilibrium and on the point of moving up the plane.
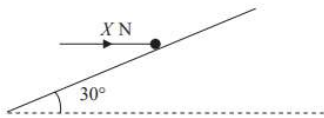
X N

30°

Figure 3

(c) Find the value of X.

(7)

?

? Diagram

Particles moving on a rough plane

Edexcel M1(Old) Jan 2010 Q5 — C

A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle,
(3)

(b) the coefficient of friction between the particle and the plane.
(5)

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 3. The particle is in equilibrium and on the point of moving up the plane.
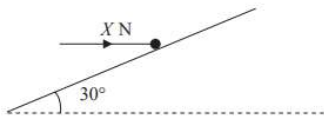
X N

30°

Figure 3

(c) Find the value of X.
(7)

To solve this part, you can either:

- $R(\uparrow) \, and \, R(\rightarrow)$

Or

- $R(\|) \, and \, R(\perp)$

?

? Diagram

Test Your Understanding

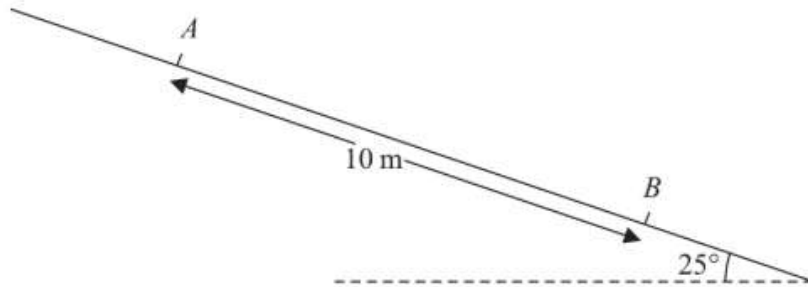**Figure 3**

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where $AB = 10$ m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B,

(b) the acceleration of P,

(c) the coefficient of friction between P and the plane.

(a) ?

(b) ?

(c) ?

Exercise 7E

Pearson Stats/Mechanics Year 2
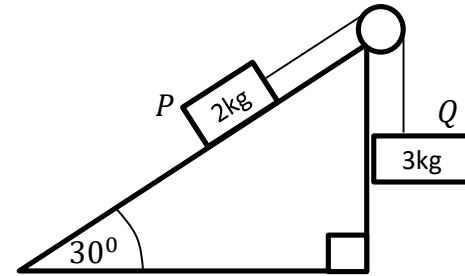Pages 148-150

Connected particles involving friction

We have already encountered problems involving connected particles in Mechanics Year 1. We just now throw friction into the mix.

Two particles P and Q, of mass 2kg and 3kg respectively, are connected by a light, inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of 30^0. Particle P is held at rest on the inclined plane and Q hangs freely on the edge of the plane with the string vertical and taut. Particle P is released and it accelerates up the plane at 2.5ms^{-2}. Find:
The tension in the string
The coefficient of friction between P and the plane
The force exerted by the string on the pulley

P 2kg

Q

3kg

30^0

? Diagram

a

?

b

?

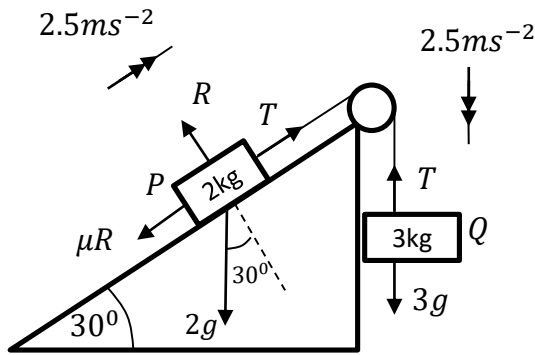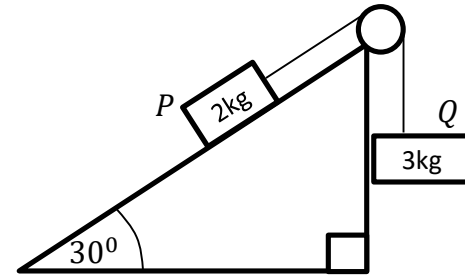# Connected particles involving friction

We have already encountered problems involving connected particles in Mechanics Year 1. We just now throw friction into the mix.

Two particles P and Q, of mass 2kg and 3kg respectively, are connected by a light, inextensible string. The string passes over a small smooth pulley which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle of 30^0. Particle P is held at rest on the inclined plane and Q hangs freely on the edge of the plane with the string vertical and taut. Particle P is released and it accelerates up the plane at 2.5ms^{-2}. Find:
The tension in the string
The coefficient of friction between P and the plane
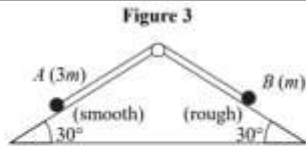The force exerted by the string on the pulley

C

?

Further Example

Figure 3

A (3m)   B (m)
(smooth)   (rough)
30°   30°

A fixed wedge has two plane faces, each inclined at 30° to the horizontal. Two particles A and B, of mass 3m and m respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a small smooth light pulley fixed at the top of the wedge. The face on which A moves is smooth. The face on which B moves is rough. The coefficient of friction between B and this face is μ. Particle A is held at rest with the string taut. The string lies in the same vertical plane as lines of greatest slope on each plane face of the wedge, as shown in Figure 3.

The particles are released from rest and start to move. Particle A moves downwards and B moves upwards. The accelerations of A and B each have magnitude $\frac{1}{18}g$.

(a) By considering the motion of A, find, in terms of m and g, the tension in the string.
(3)

(b) By considering the motion of B, find the value of μ.
(8)

(c) Find the resultant force exerted by the string on the pulley, giving its magnitude and direction.
(3)

? Diagram

a

?

b

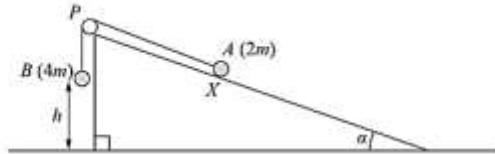?

c

?

Additional Question

7.

Figure 5

Figure 5 shows two particles A and B, of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle a, where $\tan a = \dfrac{3}{4}$. The coefficient of friction between A and the plane is $\dfrac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

(a) give a reason why the magnitudes of the accelerations of the two particles are the same,

(1)

(b) write down an equation of motion for each particle,

(4)

(c) find the acceleration of each particle.

(5)

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P. Given that A comes to rest at the point Y, without reaching P,

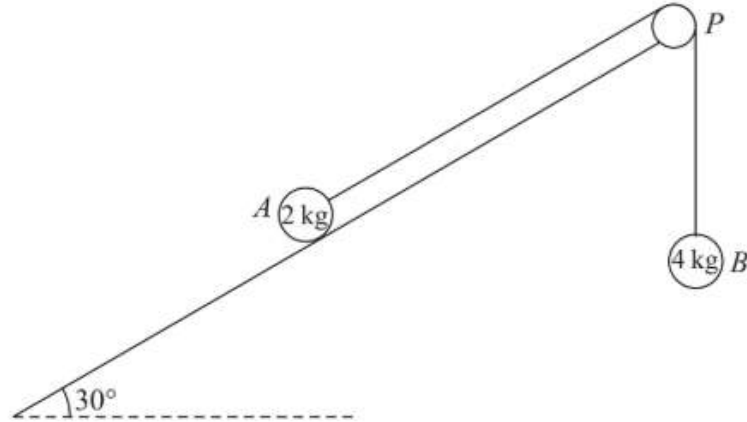(d) find the distance XY in terms of h.

(6)

?

Test Your Understanding

Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

?

Pearson Stats/Mechanics Year 2
Pages 152-154