Stats Yr2 Chapter 2 :: Conditional Probability

jfrost@tiffin.kingston.sch.uk

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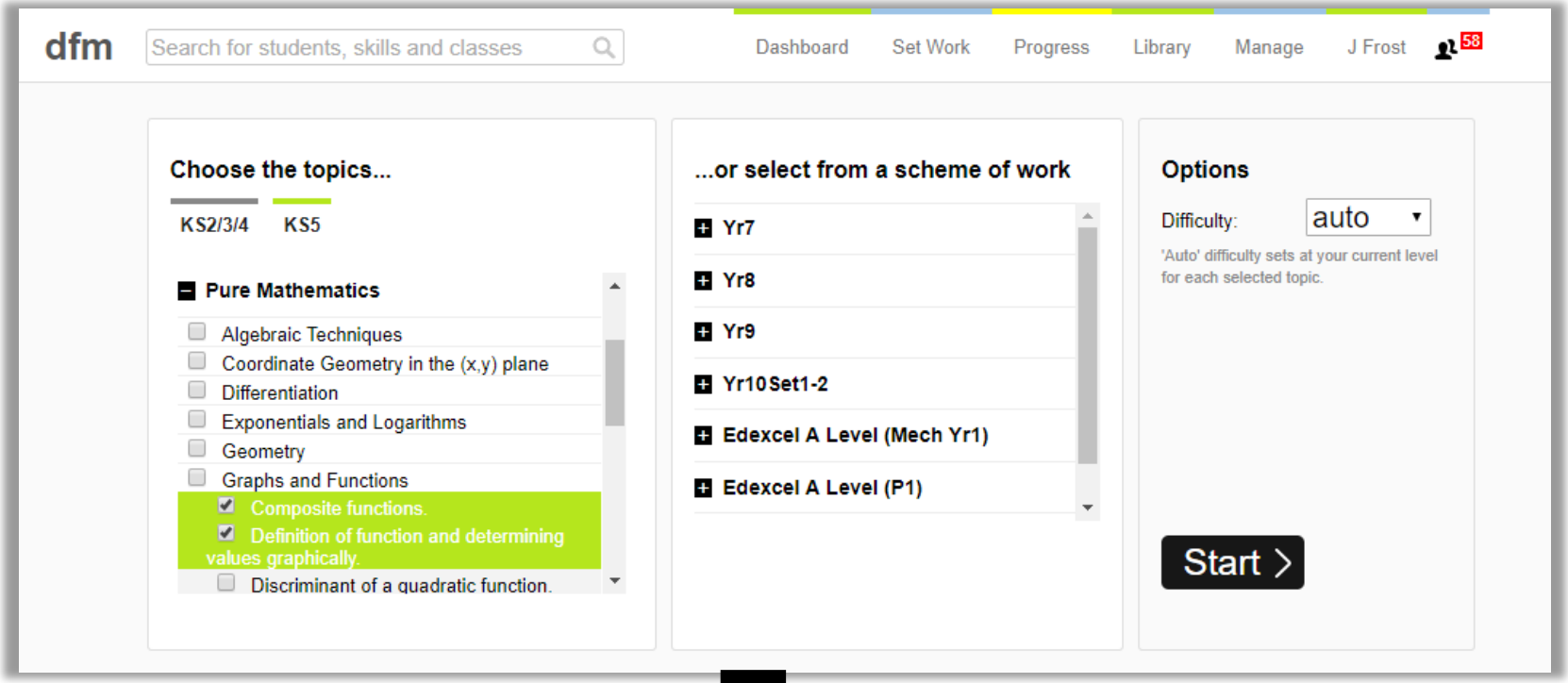

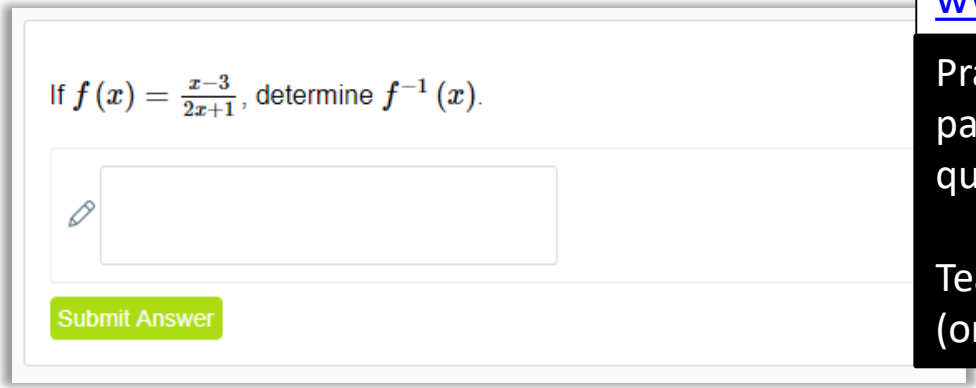

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Chapter Overview

1:: Set Notation

How sets are used to describe a sample space/event and how notation like $A \cap B$ is used to combine sets.

2:: Conditional Probability in Venn Diagrams

The notation $P(A|B)$ means "the probability of A given that B happened". How we can find such probabilities using a Venn Diagram.

3:: Formula for Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

4:: Tree Diagrams

"I have 3 red and 4 green balls in a bag. I take one ball out the bag, keep it, then take another. **Given that** the second ball was green, determine the probability the first was red."

Teacher Notes: All of this is from the old S1. The chapter was effectively split into two: all the non-conditional probability content in Year 1 and the rest in Year 2. Set notation was not used in Year 1.

RECAP :: Using sets for sample spaces and events

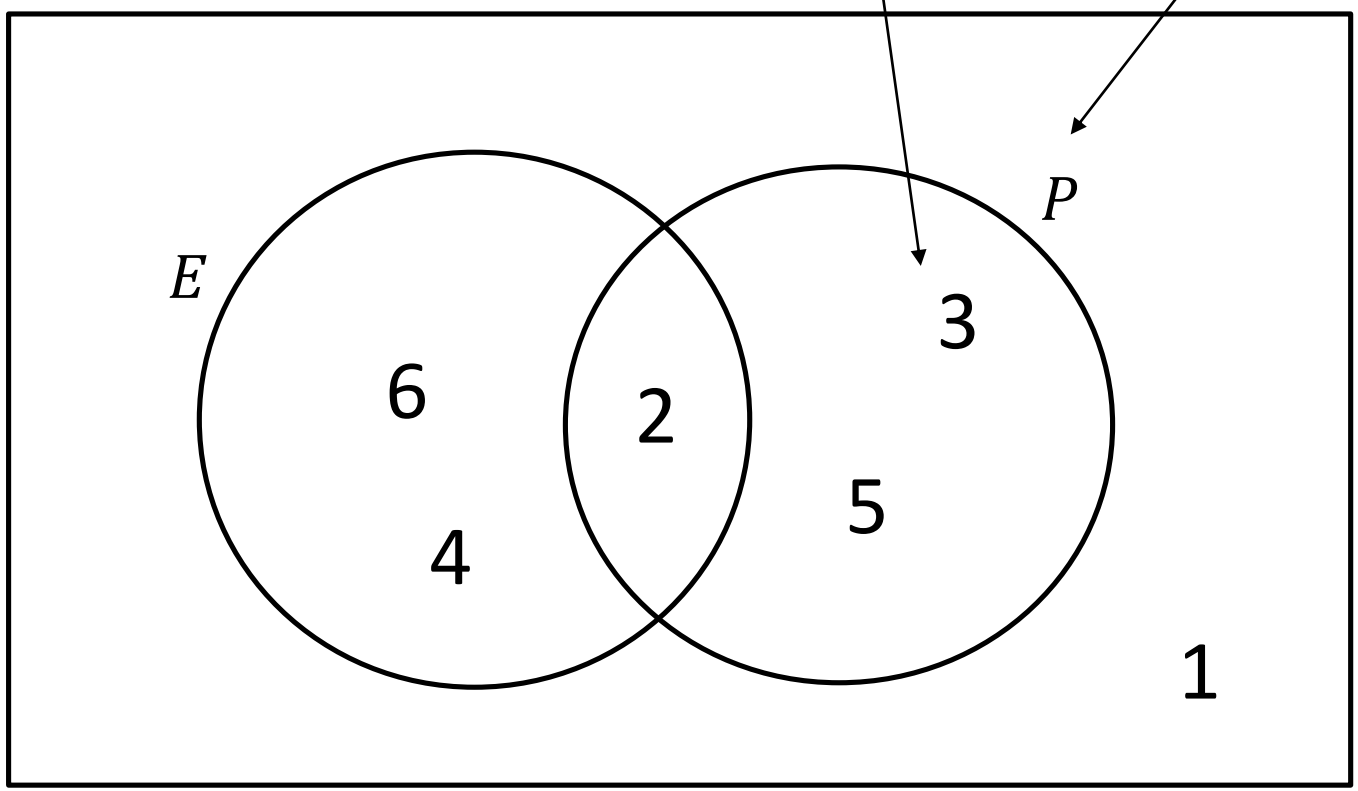
In general, sets are used to represent **collections of items**.

A **sample space** is **set of all possible outcomes**. We use ξ (Greek 'Xi'), or sometimes just S, to represent this set. We use a rectangle in a Venn Diagram.

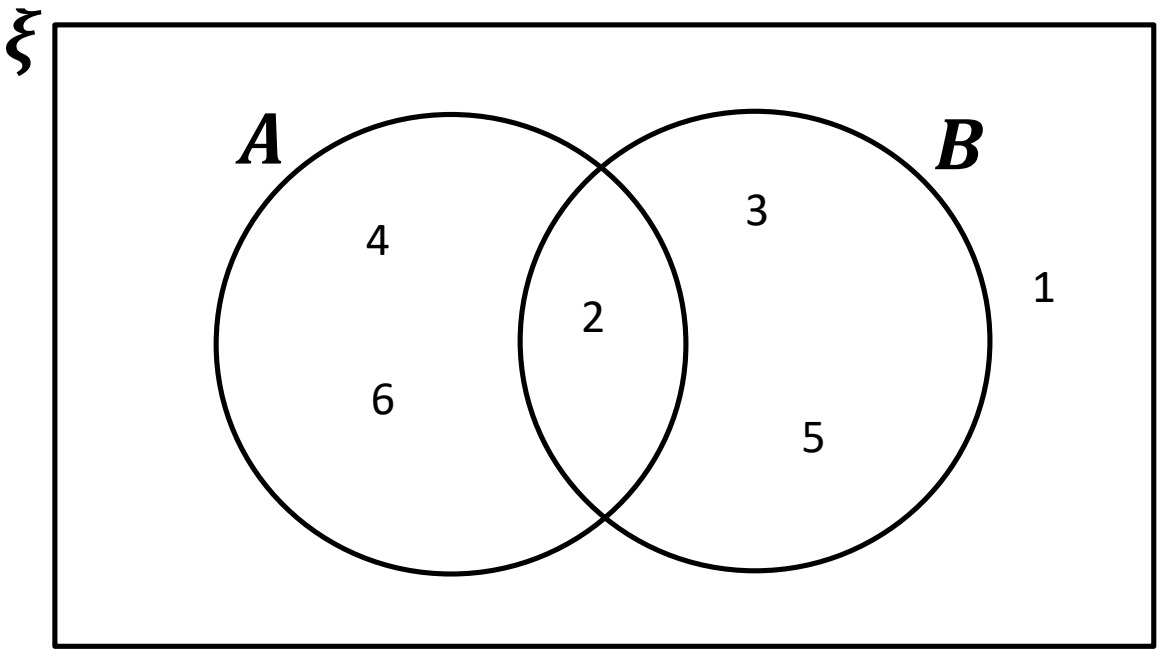
Each number represents an **outcome**.

In probability, an **event** is a **set of one or more outcomes**. These are the circles in the Venn Diagram. We use capital letters for the variables representing sets.

E = "rolling an even number"
P = "rolling a prime number"



Combining events/sets



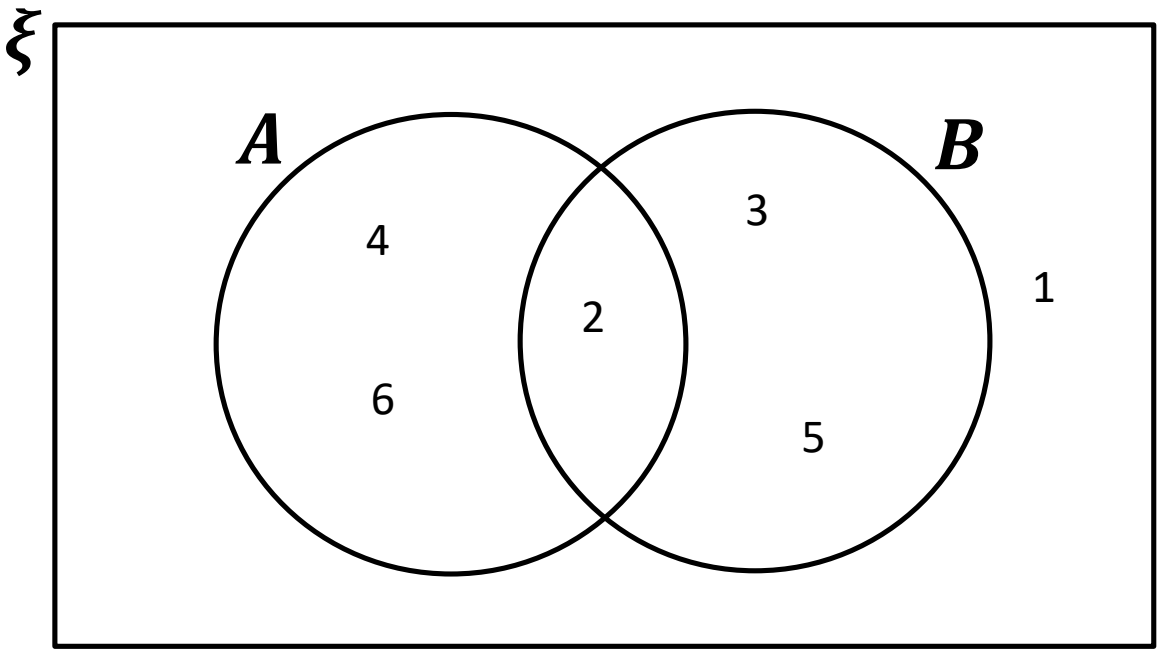
ξ = the whole sample space (1 to 6)

A = even number on a die thrown

B = prime number on a die thrown

	What does it mean in this context?	What is the resulting set of outcomes?
A'	?	?
$A \cup B$	?	?
$A \cap B$	?	?

Some fundamentals



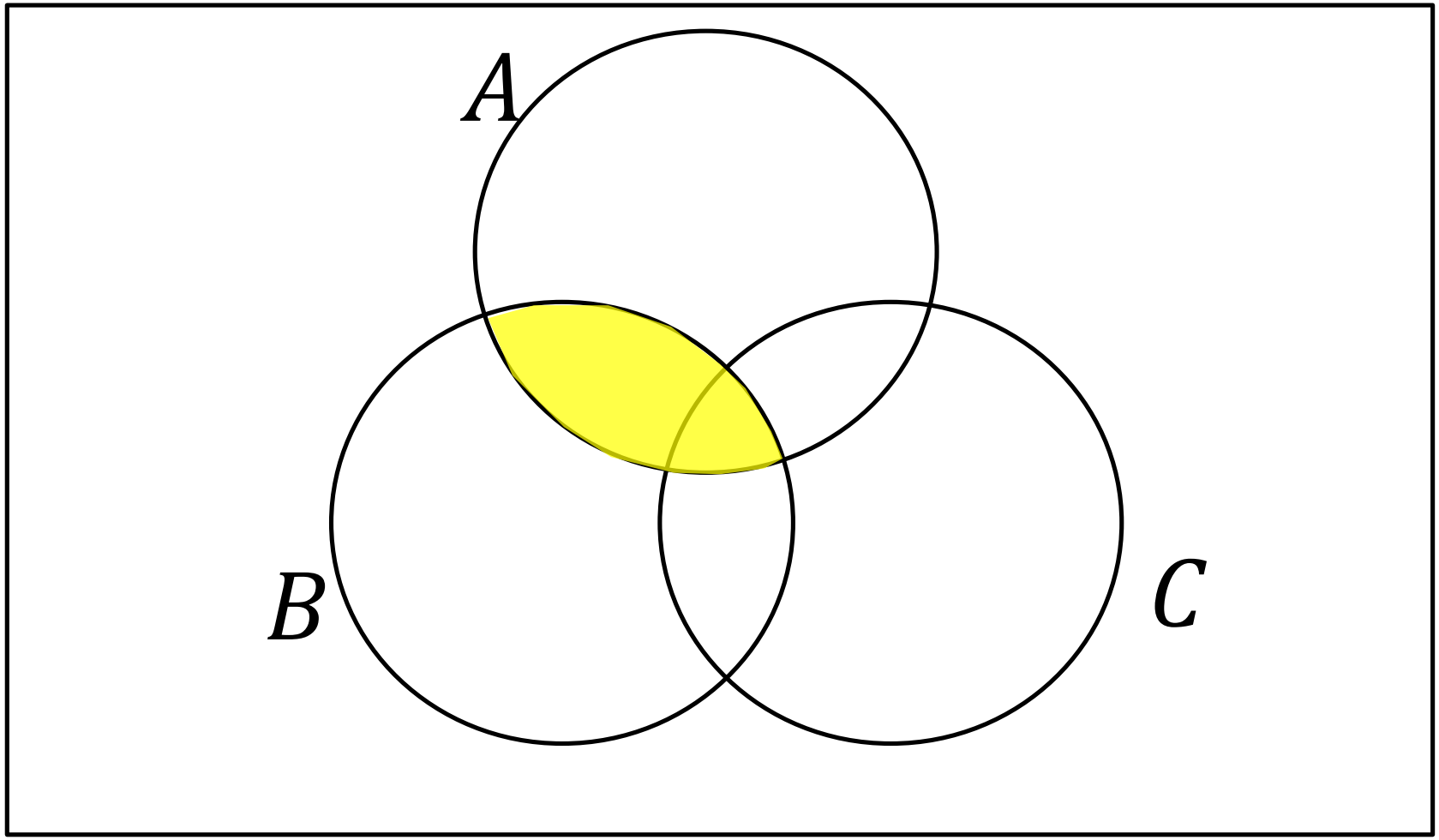
$S =$ the whole sample space (1 to 6)

$A =$ even number on a die thrown

$B =$ prime number on a die thrown

	What does it mean in this context?	What is the resulting set of outcomes?
$A \cap B'$	?	?
$(A \cup B)'$	?	?
$(A \cap B)'$	?	?

What area is indicated?



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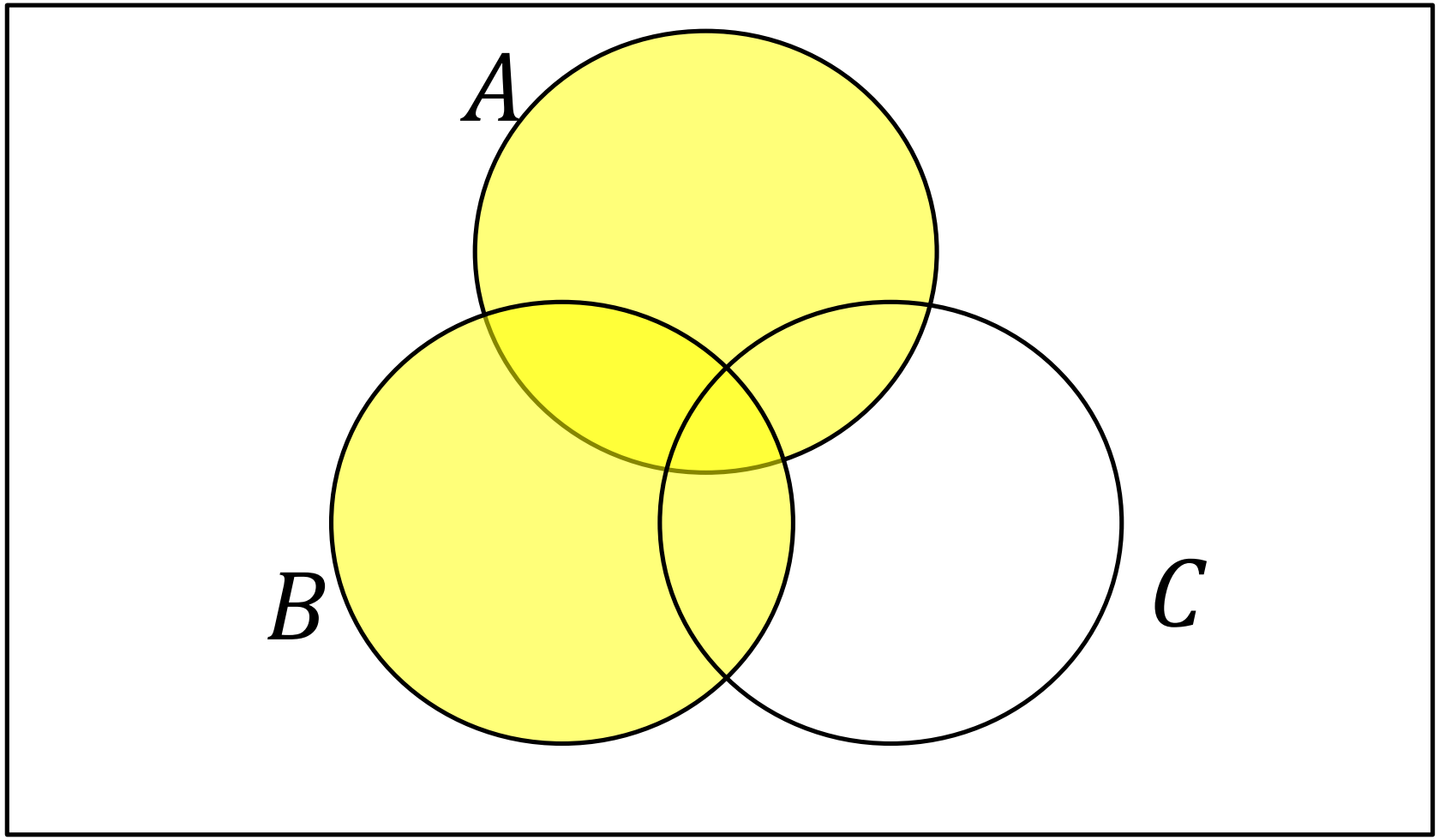
What area is indicated?



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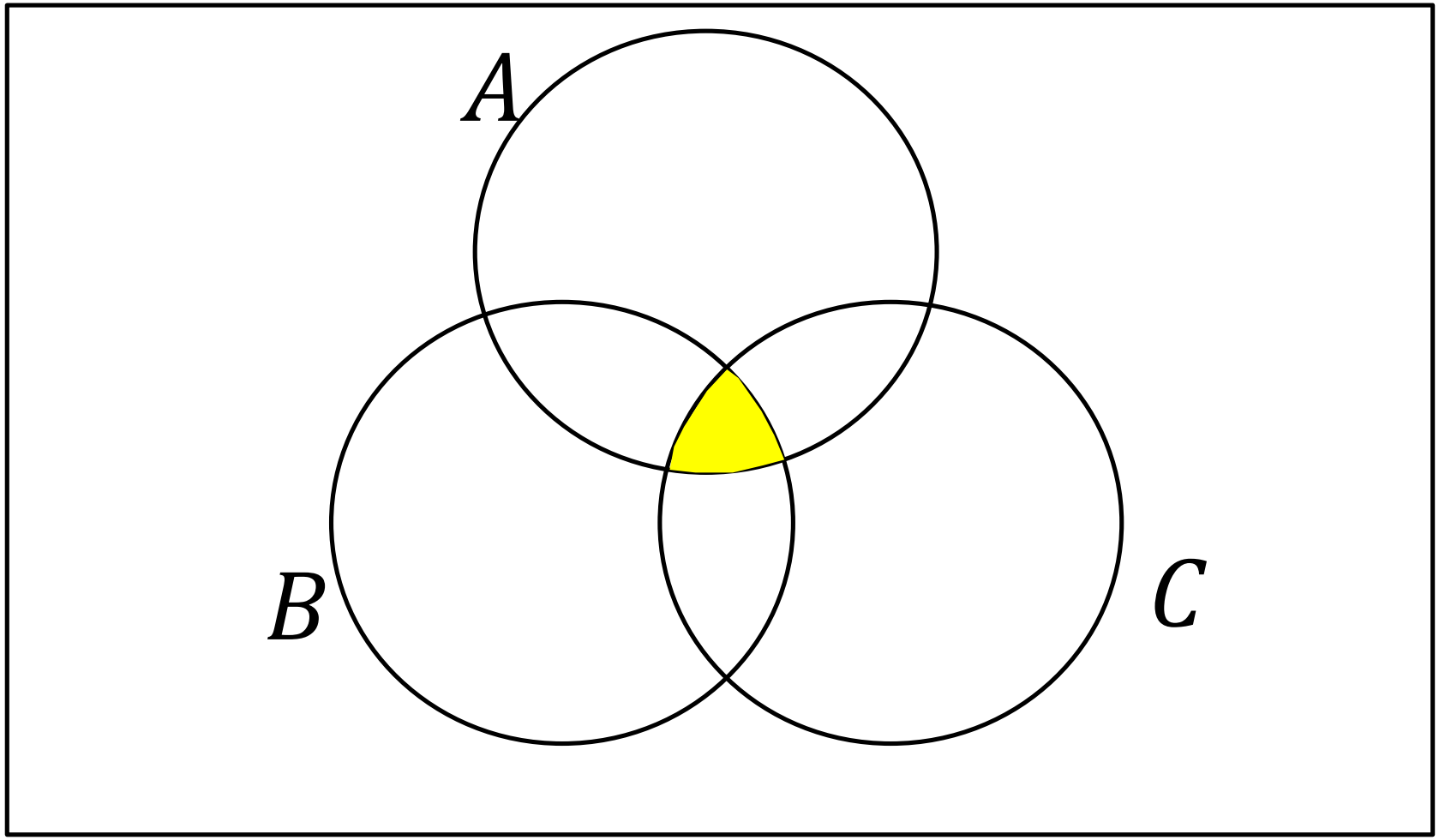

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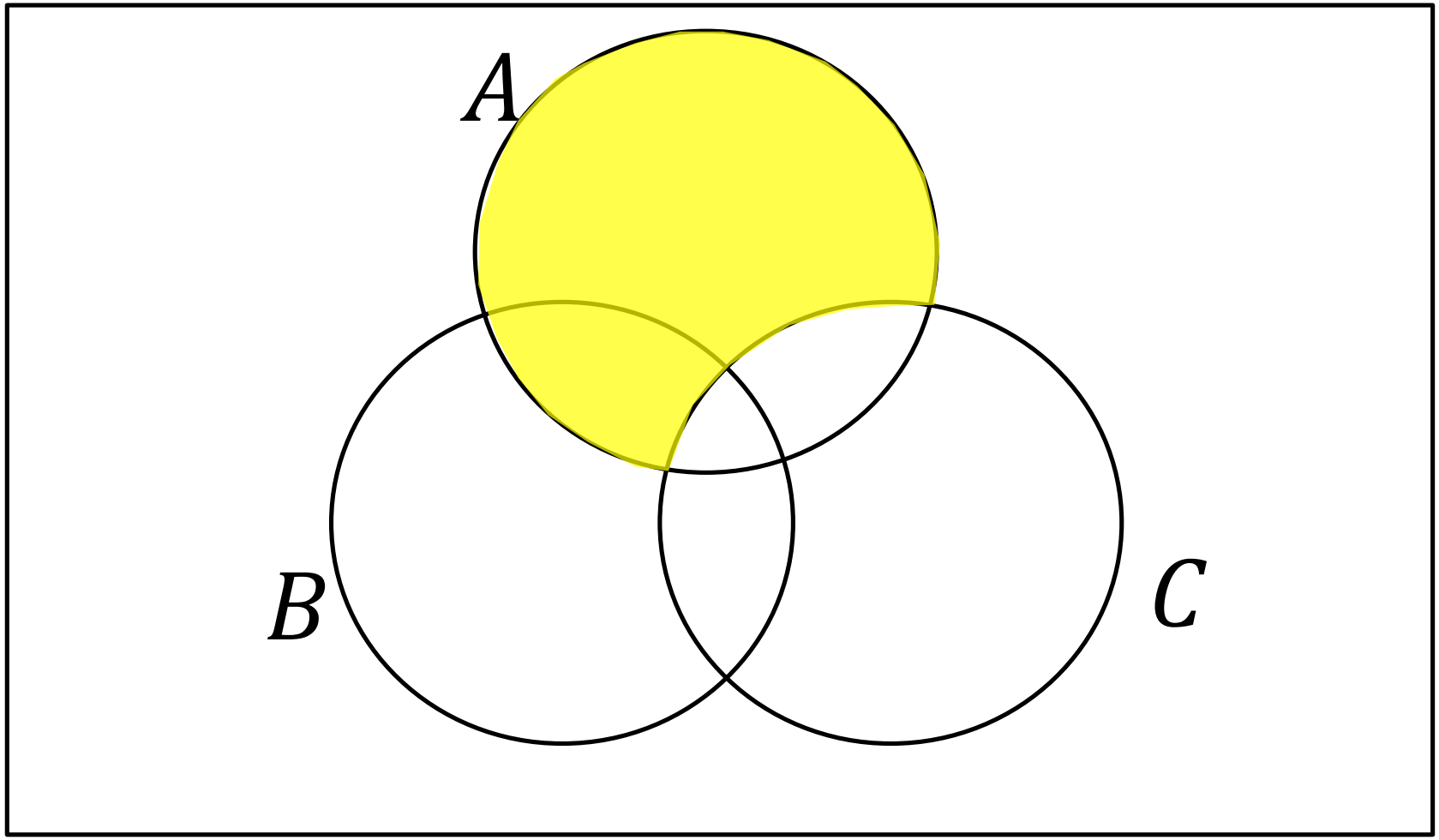
What area is indicated?



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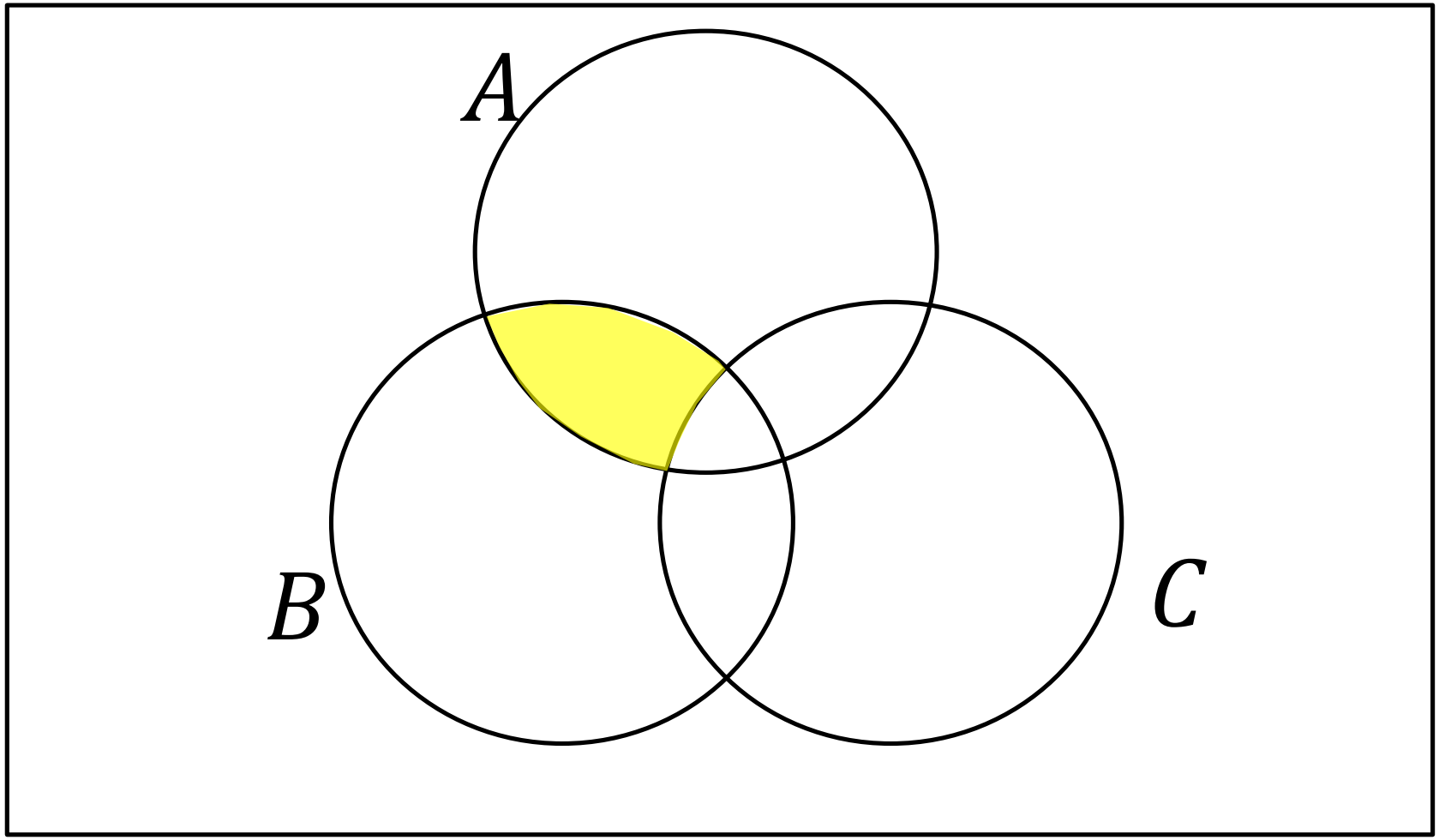

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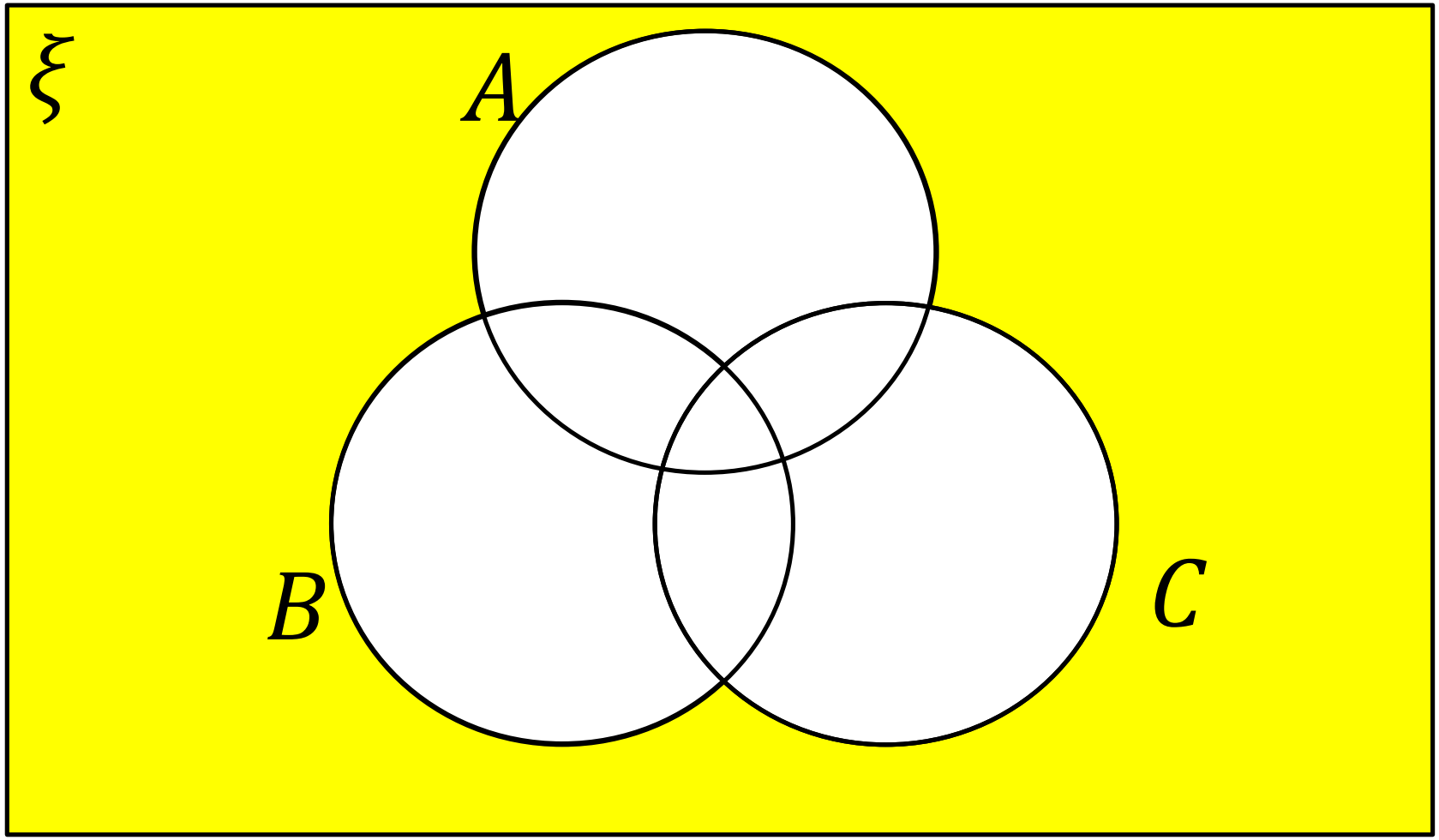
What area is indicated?

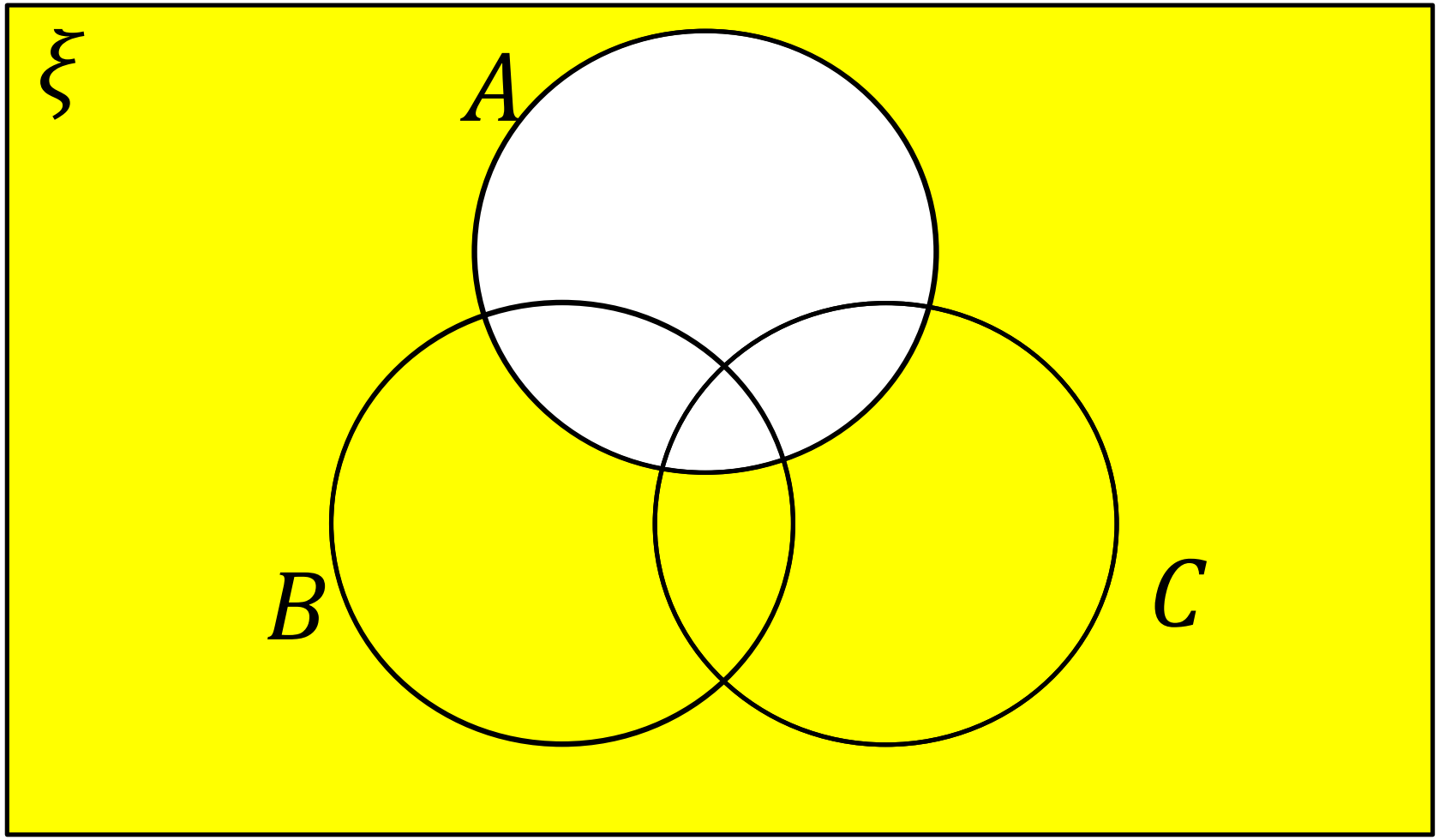


?

or
alternatively…

?

What area is indicated?



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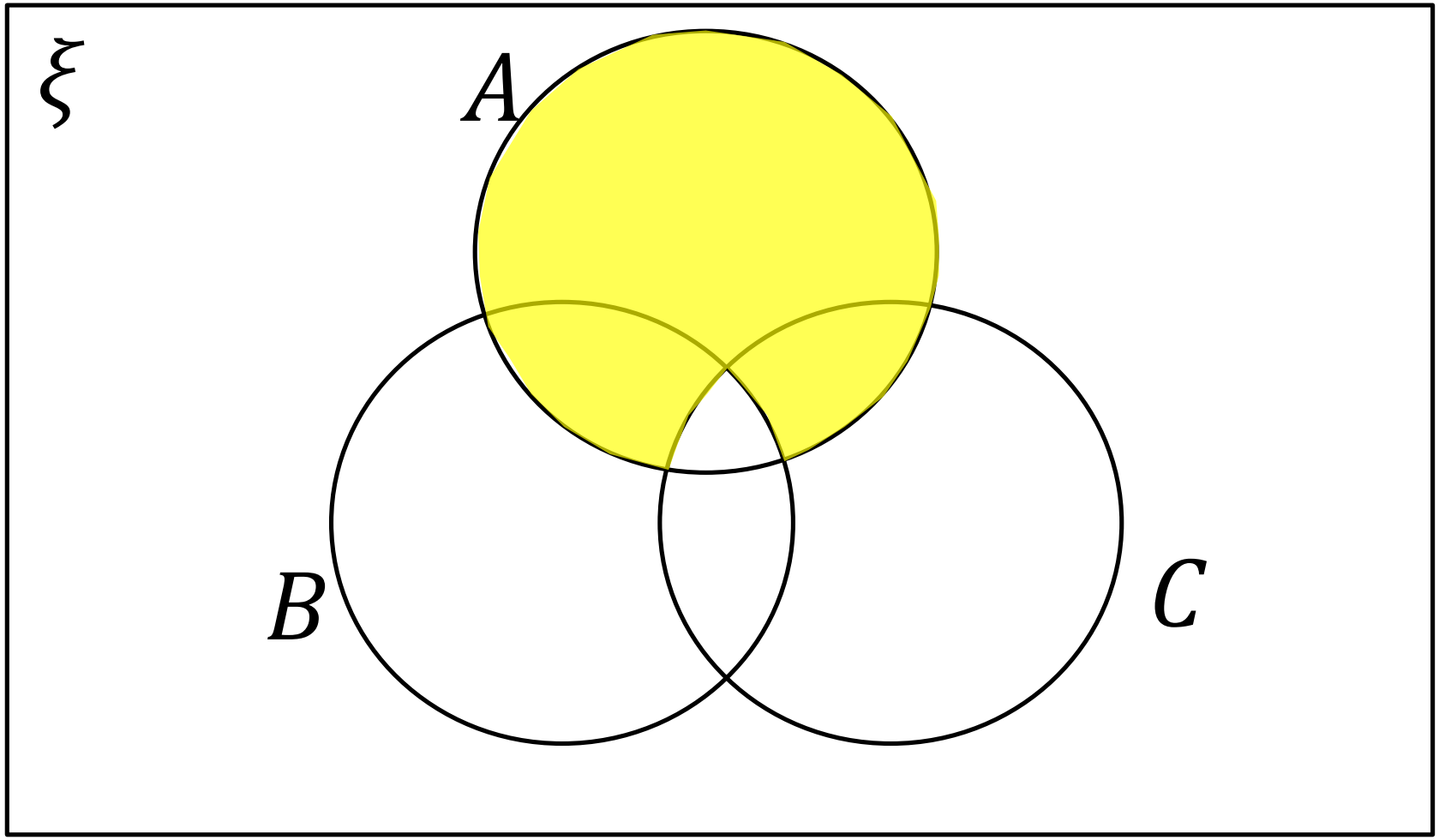

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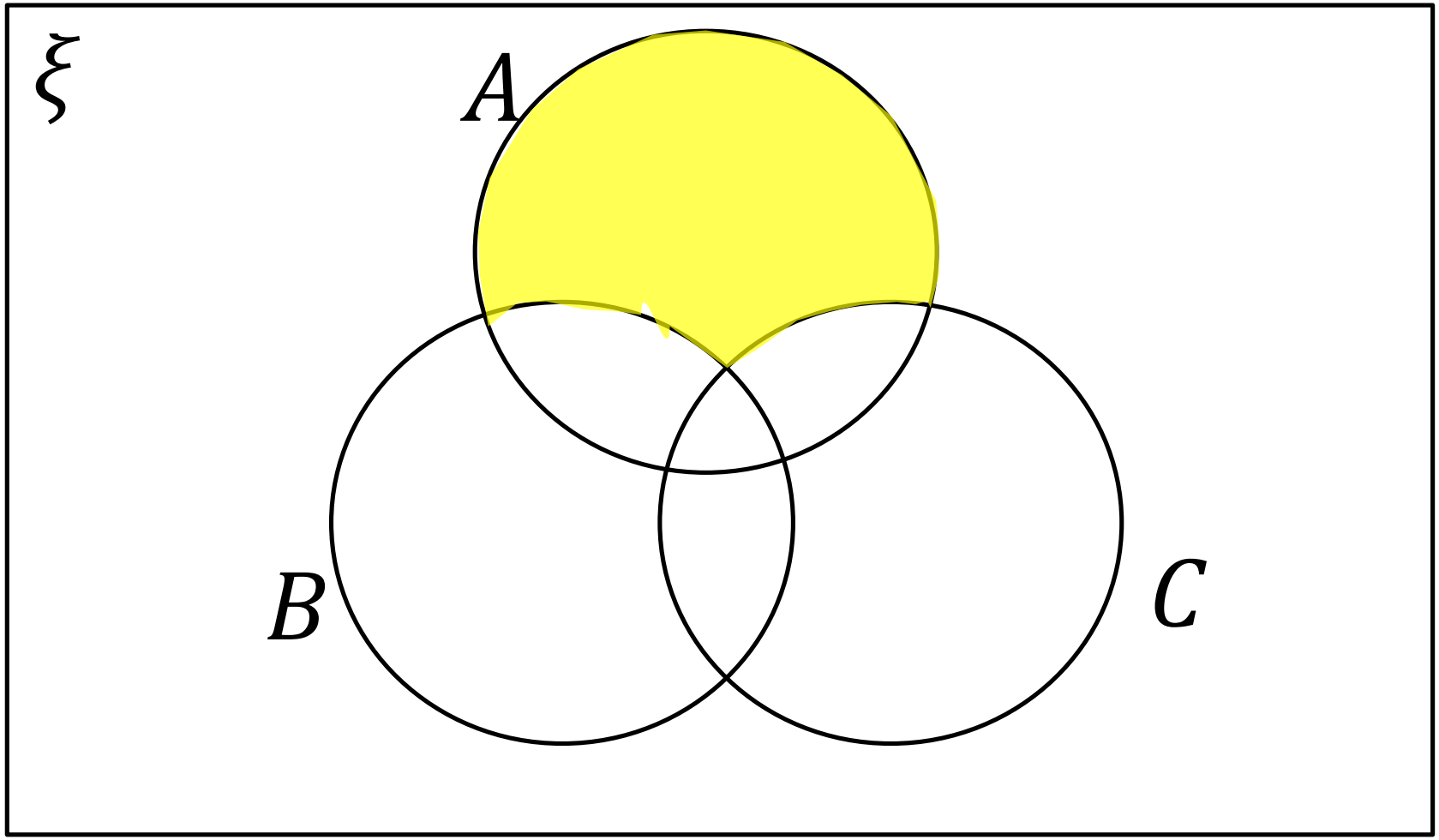


?

Examples

Venn Diagram can either contain:
(a) The **specific outcomes** in each set
(b) The number of items in the set (i.e. **frequencies**)
(c) The **probability** of being in that set.

This will usually be stated or made obvious from the context.

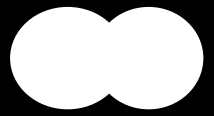
[Textbook] A card is selected at random from a pack of 52 playing cards. Let A be the event that the card is an ace and D the event that the card is a diamond. Find:
a) $P(A \cap D)$ b) $P(A \cup D)$ c) $P(A')$ d) $P(A' \cap D)$

? Venn Diagram

? a
? b
? c
? d

Think "A and D".

For union, I visualise this 'figure-of-8' shape:



Examples

[Textbook] Given that $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$,

a. Explain why events A and B are not independent.

Given also that $P(C) = 0.2$, that events A and C are mutually exclusive and that events B and C are independent,

b. Draw a Venn diagram to illustrate the events A, B and C, showing the probabilities for each region.

c. Find $P\big((A \cap B') \cup C\big)$

a

?

b

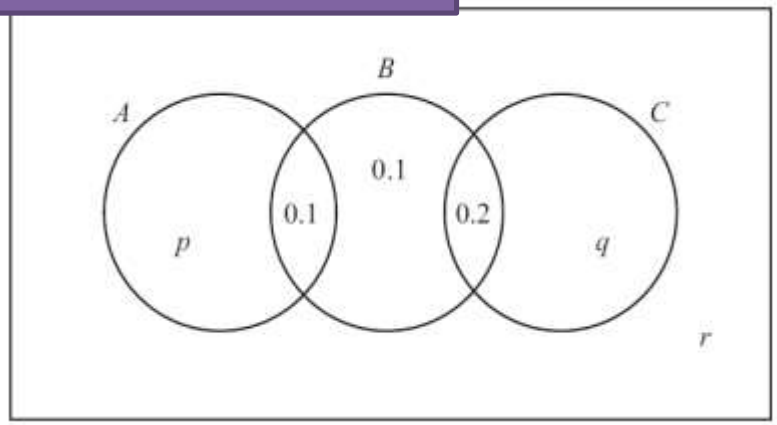
?

c

?

Test Your Understanding

May 2013 (R) Q6



The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p. **(3)**

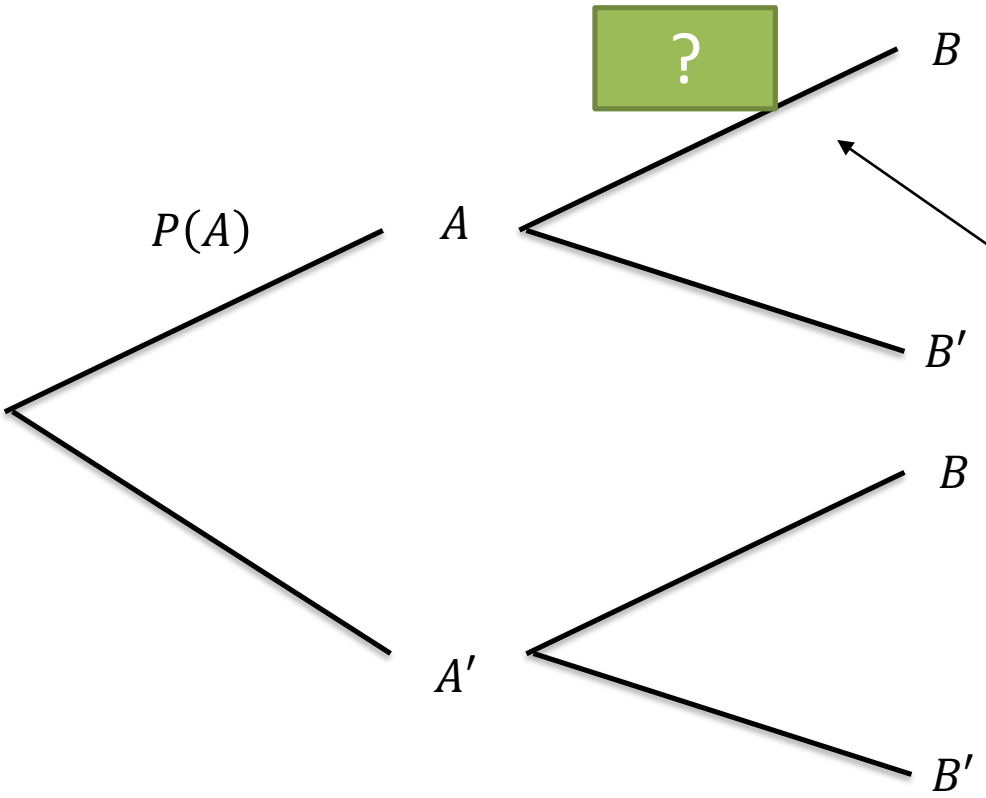
(a) ?

Exercise 2A

Pearson Stats/Mechanics Year 2
Pages 19-21

Conditional Probability

Think about how we formed a probability tree at GCSE:



$P(A \cap B)$
$=$?

Read the '|' symbol as "**given that**". i.e. "B occurred **given that** A occurred".

Alternatively (and more commonly):

$$P(B|A) = \text{?}$$

Memory Tip: You're dividing by the event you're conditioning on.

Examples

1 The following two-way table shows what foreign language students in Year 9 study.
B is the event that the student is a boy. F is the event they chose French as their language.

	B	B'	Total
F	14	38	52
F'	26	22	48
Total	40	60	100

a Determine the probability of:
$P(F|B')$

Method 1: Using the formula:

?

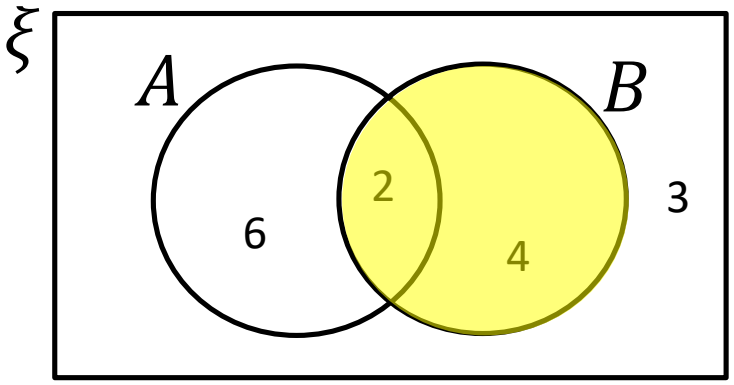
Method 2: Restricted sample space.

?

b $P(B|F') =$?

---

2 Using the Venn Diagram, determine:



a $P(A|B)$

Method 1: Using the formula

?

Method 2: Restricted sample space

?

b $P(A'|B') =$?

c $P(B|A \cup B) =$?

Further Examples

a Given that $P(A) = 0.5$ and $P(A \cap B) = 0.3$, what is $P(B|A)$?

?

Fro Tip: The 'restricted sample space' method also works for Venn Diagrams with probabilities.

b Given that $P(Y) = 0.6$ and $P(X \cap Y) = 0.4$, what is $P(X'|Y)$?
(Hint: Drawing a Venn Diagram will help!)

?

c Given that $P(A) = 0.5$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, what is $P(B|A')$?

?

Check your understanding

The events E and F are such that

$$P(E) = 0.28 \quad P(E \cup F) = 0.76 \quad P(E \cap F') = 0.11$$

Find

a) $P(E \cap F) =$?

b) $P(F) =$?

c) $P(E'|F') =$?

Click to reveal Venn Diagram

Further Practice (outside of class)

1 $P(A \cap B') = 0.4, P(A \cup B) = 0.75$

Then:

$P(B) =$?

$P(A' \cap B') =$?

2 $P(A) = 0.47$ and $P(A \cap B) = 0.12$ and $P(A' \cap B') = 0.03$

Then:

$P(A|B') =$?

3 $P(A') = 0.7, P(B') = 0.2, P(A \cap B') = 0.1$

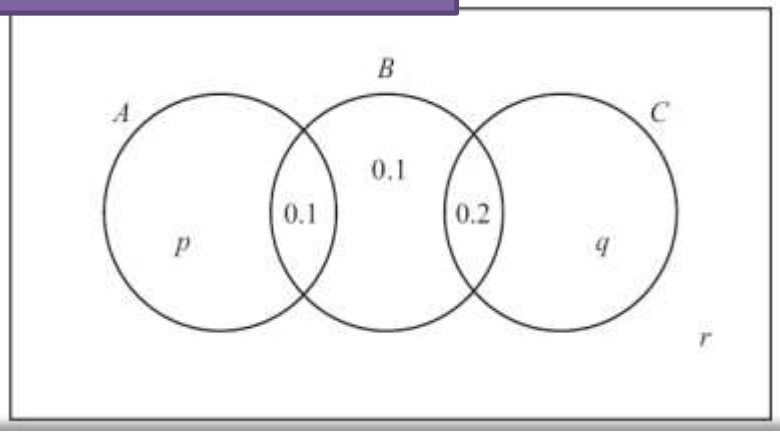
Then:

$P(A \cup B') =$?

$P(B|A') =$?

Further Test Your Understanding

May 2013 (R) Q6



The Venn diagram in Figure 1 shows three events A, B and C and the probabilities associated with each region of B. The constants p, q and r each represent probabilities associated with the three separate regions outside B.

The events A and B are independent.

(a) Find the value of p. **(3)**

Given that $P(B|C) = \frac{5}{11}$,

(b) find the value of q and the value of r (4)

(c) Find $P(A \cup C|B)$ (2)

(a) **(From earlier)**

$$0.1 = (p + 0.1) \times 0.4$$
$$p + 0.1 = 0.25$$
$$p = 0.15$$

(b) ?

(c) ?

Exercise 2C

Pearson Stats/Mechanics Year 2
Pages 25-27

Note: I have skipped Exercise 2B.

Extension:

1 [Classic puzzle] I have 2 children. One of them is a boy. What's the probability the other is a boy?

?

Full Laws of Probability

If events A and B are <u>independent</u>.

$$P(A \cap B) = \text{?}$$
$$P(A|B) = \text{?}$$

If events A and B are <u>mutually exclusive</u>:

$$P(A \cap B) = \text{?}$$
$$P(A \cup B) = \text{?}$$

In general:

We first encountered this in the previous section.

$$P(A|B) = \text{?}$$

This is known as the **<u>Addition Law</u>.** **Informal Proof**: If we added the probabilities in the A and B sets in the Venn Diagram, we'd be double counting the intersection, so subtract so that it's only counted once.

$$P(A \cup B) = \text{?}$$

Example

Edexcel S1

6. Explain what you understand by

(a) a sample space, **(1)**

(b) an event. **(1)**

Two events A and B are independent, such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

Find

(c) $P(A \cap B)$, **(1)**

(d) $P(A \mid B)$, **(2)**

(e) $P(A \cup B)$. **(2)**

a) ?

b) ?

c) ?

d) ?

e) ?

Further Examples

[Textbook] C and D are two events such that $P(C) = 0.2$, $P(D) = 0.6$ and $P(C|D) = 0.3$. Find:
a. $P(C \cap D)$ b. $P(D|C)$ c. $P(C \cup D)$

? a

? b

? c

Hints for (b): You saw the words "are independent". So write out $P(A \cap B) = P(A)P(B)$.
Also, you're given $P(A \cup B)$ which suggests you might be able to use the Addition Rule.

10. [Jan 2012 Q2] (a) State in words the relationship between two events R and S when $P(R \cap S) = 0$. **(1)**

The events A and B are independent with $P(A) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$. Find

(*b*) $P(B)$, **(4)**
(*c*) $P(A' \cap B)$, **(2)**
(*d*) $P(B'|A)$. **(2)**

(a) ?

(b) ?

(c) ?

(d) ?

Test Your Understanding

Edexcel S1

9. Three events A, B and C are defined in the sample space S. The events A and B are mutually exclusive and A and C are independent.

(a) Draw a Venn diagram to illustrate the relationships between the 3 events and the sample space. **(3)**

Given that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup C) = 0.7$, find

(b) $P(A \mid C)$, **(2)**

(c) $P(A \cup B)$, **(2)**

(d) $P(C)$. **(4)**

a)

?

b)

?

c)

?

d)

?

SUPER IMPORTANT TIPS

If I were to identify two tips that will possible help you the most in probability questions:

If you see the words '**given that**', Immediately write out the law for conditional probability.

Example: "Given Bob walks to school, find the probability that he's not late…"

First thing you should write: ?

If you see the words '**are independent**', Immediately write out the laws for independence.
(Even before you've finished reading the question!)

Example: "A is independent from B…"

First thing you should write: ?

If you're stuck on a question where you have to find a probability given others, it's probably because you've failed to take into account that two events are independent or mutually exclusive, or you need to use the conditional probability or additional law.

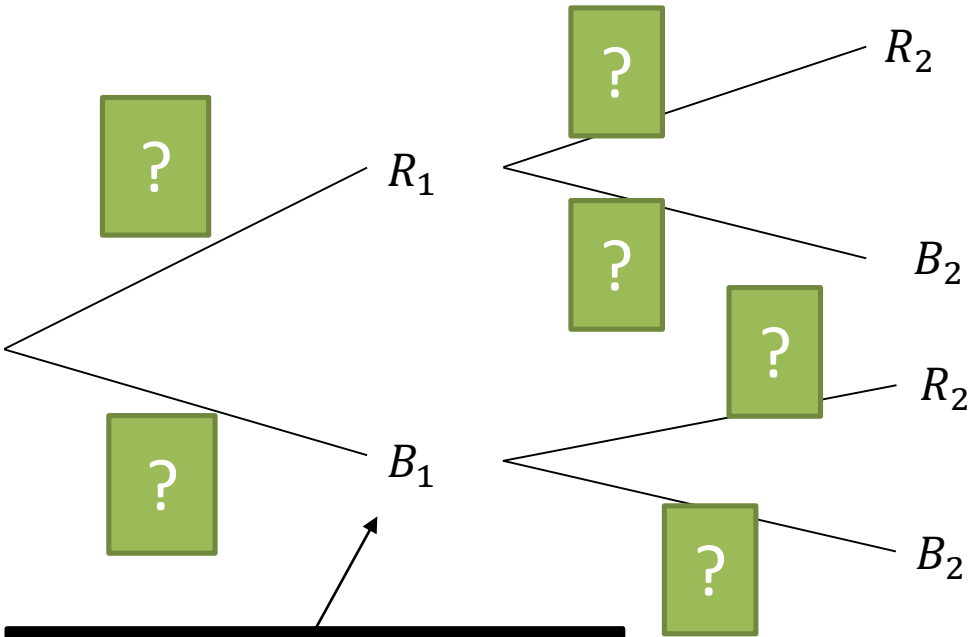
Exercise 2D

Pearson Stats/Mechanics Year 2
Pages 29-30

Probability Trees

We saw probability trees in Year 1. The only difference here is **determining a conditional probability** using your tree.

Example: You have two bags, the first with 5 red balls and 5 blue balls, and the second with 3 red balls and 6 blue balls. You first pick a ball from the first bag, and place it in the second. You then pick a ball from the second bag. Complete the tree diagram.



Fro Tip: Use variable subscripting to indicate what pick you're referring to.

Hence find the probability that:

a) You pick a red ball on your second pick.

?

b) Given that your second pick was red, the first pick was also red.

?

Further Example

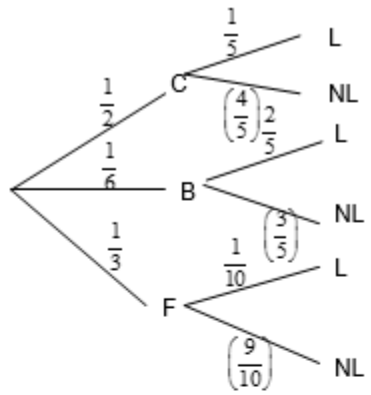
Edexcel S1 May 2009 Q2

On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}, \frac{2}{5}$ and $\frac{1}{10}$ respectively.

(*c*) Given that Bill is late, find the probability that he did not travel on foot. **(4)**

(Part (a) asks for a tree diagram, which may help with this question)

(a)



Correct tree All labels Probabilities on correct branches	B1 B1 B1

?

Testing Your Understanding

Edexcel S1

6. **[Jan 2006 Q4]** A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

(*a*) Draw a tree diagram to represent the information. **(3)**

Find the probability that

(*a*) the second ball selected is red, **(2)**

(*b*) both balls selected are red, given that the second ball selected is red. **(2)**

(a) ?

(b) ?

(c) ?

Exercise 2E

Pearson Stats/Mechanics Year 2
Pages 31-34