# Quadratic inequalities

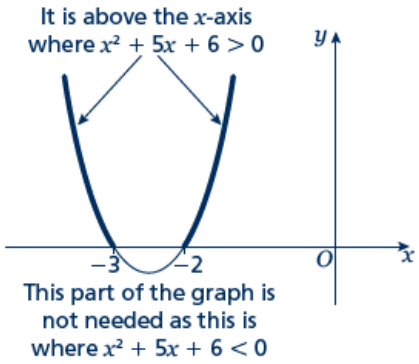
**A LEVEL LINKS**
**Scheme of work:** 1d. Inequalities – linear and quadratic (including graphical solutions)

## Key points

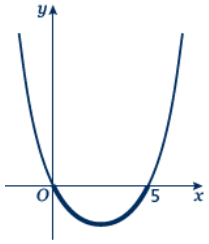
- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

## Examples

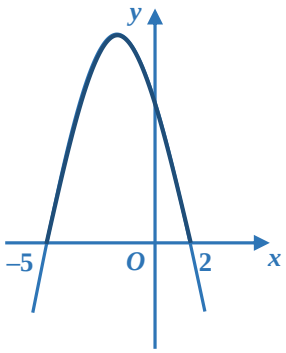
**Example 1** Find the set of values of $x$ which satisfy $x^2 + 5x + 6 > 0$

| | |
|---|---|
| $x^2 + 5x + 6 = 0$<br>$(x + 3)(x + 2) = 0$<br>$x = -3$ or $x = -2$<br>  <br><br>$x < -3$ or $x > -2$ | **1** Solve the quadratic equation by factorising.<br>**2** Sketch the graph of $y = (x + 3)(x + 2)$<br>**3** Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$<br>**4** Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$ |

**Example 2** Find the set of values of $x$ which satisfy $x^2 - 5x \leq 0$

| | |
|---|---|
| $x^2 - 5x = 0$<br>$x(x - 5) = 0$<br>$x = 0$ or $x = 5$<br>  <br><br>$0 \leq x \leq 5$ | **1** Solve the quadratic equation by factorising.<br>**2** Sketch the graph of $y = x(x - 5)$<br>**3** Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$<br>**4** Write down the values which satisfy the inequality $x^2 - 5x \leq 0$ |

**Example 3** Find the set of values of $x$ which satisfy $-x^2 - 3x + 10 \geq 0$

| | |
|---|---|
| $-x^2 - 3x + 10 = 0$<br>$(-x + 2)(x + 5) = 0$<br>$x = 2$ or $x = -5$<br>  <br><br>$-5 \leq x \leq 2$ | **1** Solve the quadratic equation by factorising.<br><br>**2** Sketch the graph of $y = (-x + 2)(x + 5) = 0$<br><br>**3** Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$<br><br>**3** Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$ |

## Practice

**1** Find the set of values of $x$ for which $(x + 7)(x - 4) \leq 0$

**2** Find the set of values of $x$ for which $x^2 - 4x - 12 \geq 0$

**3** Find the set of values of $x$ for which $2x^2 - 7x + 3 < 0$

**4** Find the set of values of $x$ for which $4x^2 + 4x - 3 > 0$

**5** Find the set of values of $x$ for which $12 + x - x^2 \geq 0$

## Extend

Find the set of values which satisfy the following inequalities.

**6** $x^2 + x \leq 6$

**7** $x(2x - 9) < -10$

**8** $6x^2 \geq 15 + x$

# Answers

**1** $-7 \leq x \leq 4$

**2** $x \leq -2$ or $x \geq 6$

**3** $\frac{1}{2} < x < 3$

**4** $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

**5** $-3 \leq x \leq 4$

**6** $-3 \leq x \leq 2$

**7** $2 < x < 2\frac{1}{2}$

**8** $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$