Linear Graphs

Chapter Overview

1. $y = mx + c$

2. Parallel and perpendicular lines

3. Lengths and Areas

4. Modelling

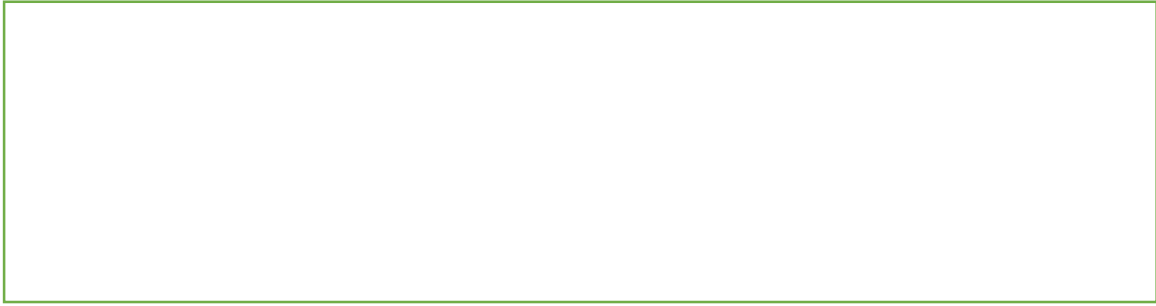| 3.1	Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$;	To include the equation of a line through two given points, and the equation of a line parallel (or perpendicular) to a given line through a given point.
	Gradient conditions for two straight lines to be parallel or perpendicular.	$m' = m$ for parallel lines and $m' = -\dfrac{1}{m}$ for perpendicular lines
	Be able to use straight line models in a variety of contexts.	For example, the line for converting degrees Celsius to degrees Fahrenheit, distance against time for constant speed, etc.

1. Linear Graphs

Examples:

1. The point $(5, a)$ lies on the line with equation $y = 3x + 2$. Determine the value of a.

2. Find the coordinate of the point where the line $2x + y = 5$ cuts the x-axis.

Test Your Understanding:

Determine where the line $x + 2y = 3$ crosses both the axes

Gradient

Examples:

Find the gradient of the line between the following sets of points:
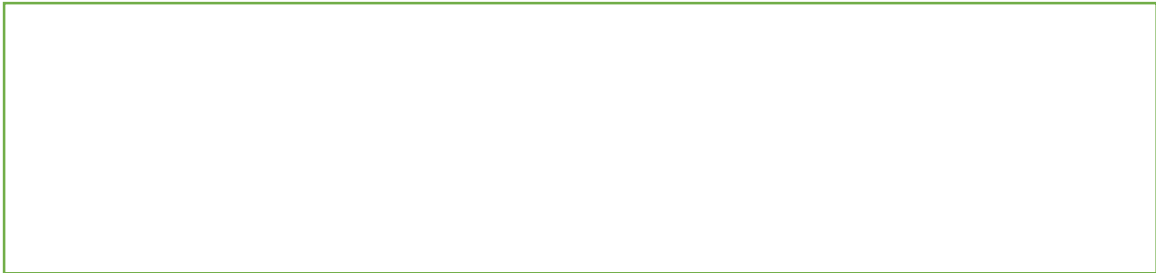
1. $(1, 4)$ $(3, 10)$

2. $(5, 7)$ $(8, 1)$

3. $(2, 2)$ $(-1, 10)$

4. Show that the points $A(3,4)$, $B(5,5)$, $C(11,8)$ all lie on a straight line.

5. The line joining $(2, -5)$ to $(4, a)$ has gradient -1. Work out the value of a.
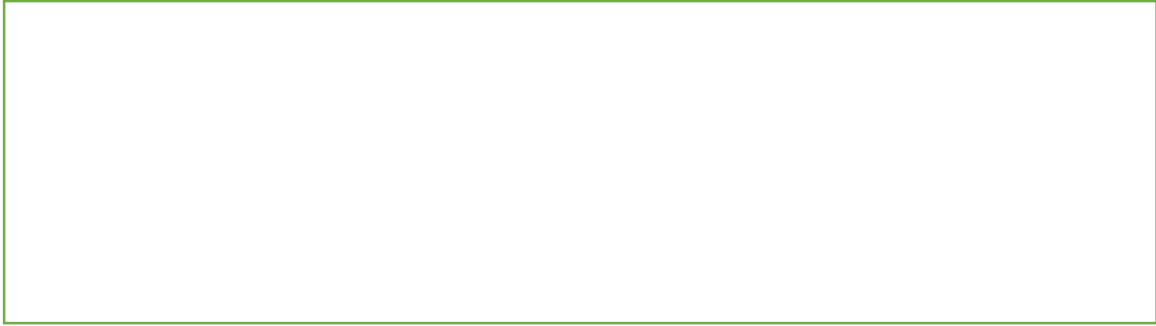
$\underline{y = mx + c}$

Example:

Determine the gradient and y-intercept of the line with equation $4x - 3y + 5 = 0$

$ax + by + c = 0$

Example

Express $y = \frac{1}{3}x - \frac{2}{3}$ in the form $ax + by + c = 0$, where a, b, c are integers.

Test Your Understanding

Express $y = \frac{2}{5}x + \frac{3}{5}$ in the form $ax + by + c = 0$, where a, b, c are integers.
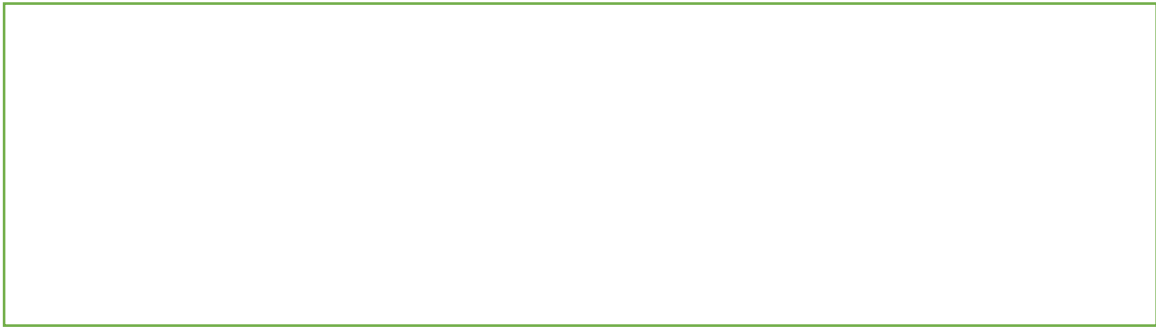
Equations using one point and the gradient

<div style="border:1px solid green; min-height:320px;"></div>

Example

Find the equation of the line that goes through $(3,5)$ and has gradient 2.

Quickfire Questions

Gradient	Point	(Unsimplified) Equation
3	$(1,2)$	
5	$(3,0)$	
2	$(-3,4)$	
$\dfrac{1}{2}$	$(1,-5)$	
9	$(-4,-4)$	

Finding a line using 2 Points:

Example

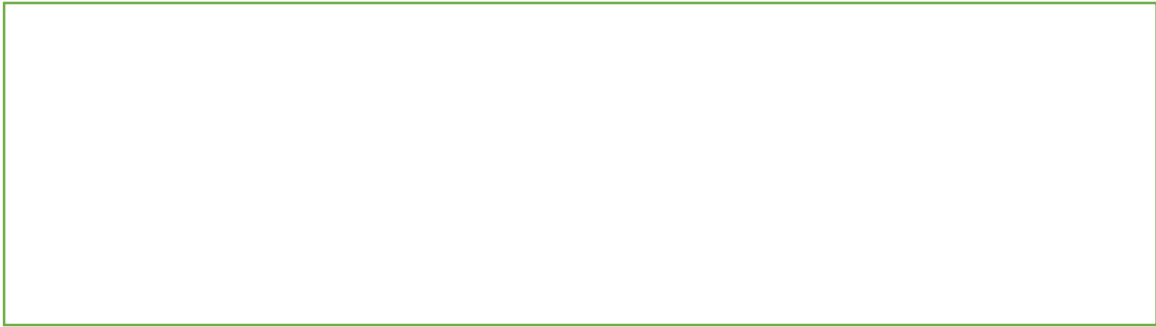1. Find the equation of the line that goes through $(4,5)$ and $(6,2)$, giving your equation in the form
$$ax + by + c = 0.$$

Test Your Understanding:

1. Find the equation of the line that goes through $(-1,9)$ and $(4,5)$, giving your equation in the form
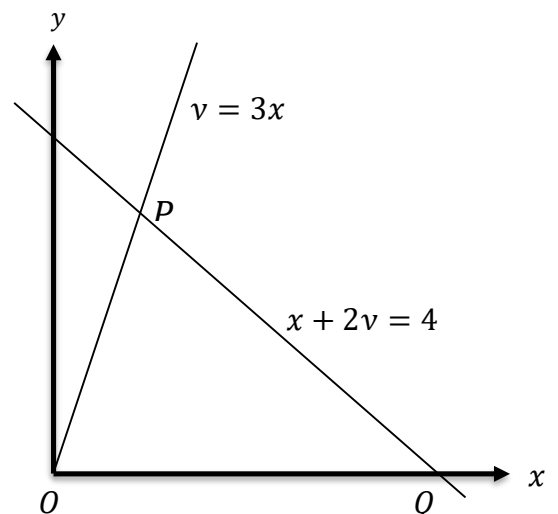$$ax + by + c = 0.$$

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Intersection of Lines:

Example

The diagram shows two lines with equations $y = 3x$ and $x + 2y = 4$, which intersect at the point P.

a. Determine the coordinates of P.

b. The line $x + 2y = 4$ intersects the x-axis at the point Q. Determine the coordinate of Q.

Test Your Understanding

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2 .

(3)

Perpendicular Lines

Quickfire Questions

Gradient	Gradient of Perpendicular Line
2	
-3	
$\dfrac{1}{4}$	
5	
$-\dfrac{2}{7}$	
$\dfrac{7}{5}$	

Problems

1. A line is goes through the point (9,10) and is perpendicular to another line with equation $y = 3x + 2$. What is the equation of the line?

2. A line L_1 goes through the points $A(1,3)$ and $B(3,-1)$. A second line L_2 is perpendicular to L_1 and passes through point B. Where does L_2 cross the x-axis?

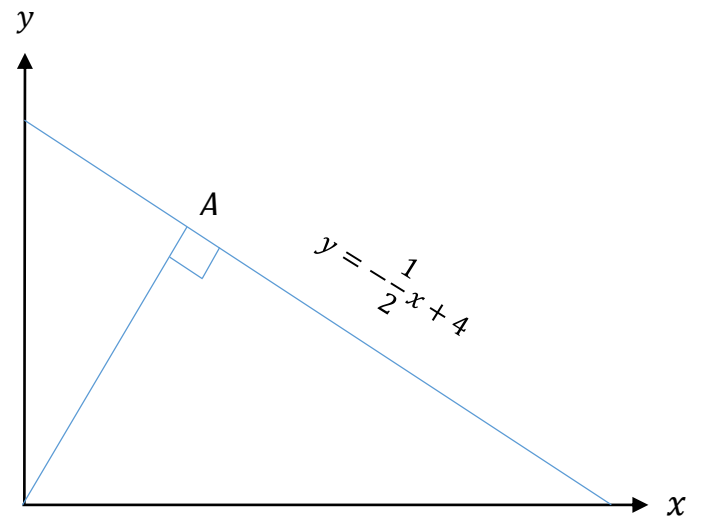3. Are the following lines parallel, perpendicular, or neither?

$$y = \frac{1}{2}x$$

$$2x - y + 4 = 0$$

Test Your Understanding

1. A line goes through the point (4,7) and is perpendicular to another line with equation $y = 2x + 2$. What is the equation of the line? Put your answer in the form $ax + by + c = 0$, where a, b, c are integers.

2. Determine the point A.

Extension

1. *[MAT 2004 1D]*
What is the reflection of the point $(3,4)$ in the line $3x + 4y = 50$?

2. *[MAT 2014 1D]* The reflection of the point $(1,0)$ in the line $y = mx$ has coordinates: (in terms of m)

3. *[STEP I 2004 Q6]* The three points A, B, C have coordinates $(p_1, q_1), (p_2, q_2)$ and (p_3, q_3), respectively. Find the point of intersection of the line joining A to the midpoint of BC, and the line joining B to the midpoint of AC. Verify that this point lies on the line joining C to the midpoint of AB.

The point H has coordinates $(p_1 + p_2 + p_3, q_1 + q_2 + q_3)$. Show that if the line AH intersects the line BC at right angles, then $p_2^2 + q_2^2 = p_3^2 + q_3^2$, and write down a similar result if the line BH intersects the line AC at right angles.

Deduce that if AH is perpendicular to BC and also BH is perpendicular to AC, then CH is perpendicular to AB.

Distances between points
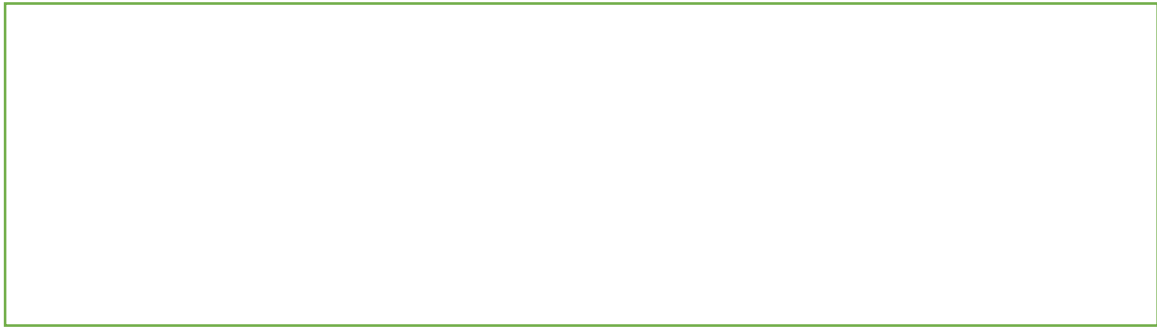
Examples

Find the distance between

$(3,4)$ and $(5,7)$

$(5,1)$ and $(6,-3)$

$(0,-2)$ and $(-1,3)$

Test Your Understanding

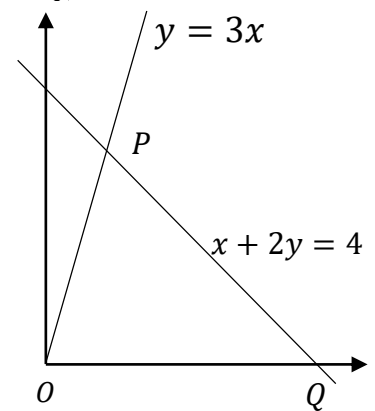**Find the distance between:**

$(1,10)$ and $(4,14)$

$(3,-1)$ and $(0,1)$

$(-4,-2)$ and $(-12,4)$

Area of Shapes

Example 1

The diagram shows two lines with equations $y = 3x$

and $x + 2y = 4$, which intersect at the point P.
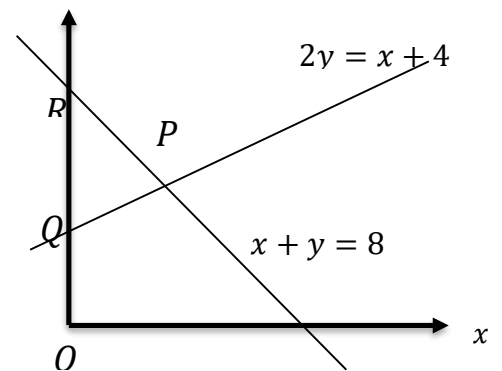
a)Determine the coordinates of P.

b) The line $x + 2y = 4$ intersects the x-axis at the point Q. Determine the area of the triangle OPQ.

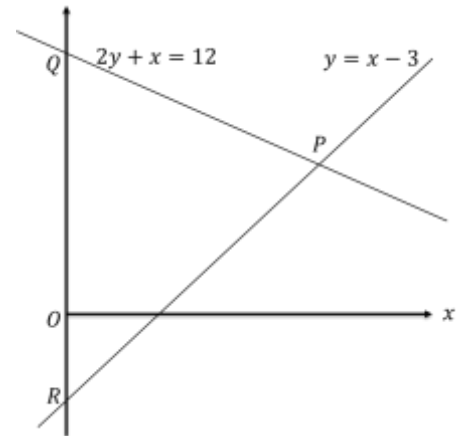When $y = 0$, $x = 4$

Example 2

a)Determine the length of PQ

b) Determine the area PQR.

Test Your Understanding:

a) Determine the coordinate of P.

b) Determine the area of PQR.

c) Determine the length PQ.

Extension
[MAT 2001 1C]
The shortest distance from the origin to the line $3x + 4y = 25$ is what?

Exercise 5G Page
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Modelling with Linear Graphs

Many real life variables have a 'linear' relationship, i.e. there is a fixed increase/decrease in one variable each time the other variable goes up by 1 unit.

Example

The temperature y at different points on a mountain is recorded at different altitudes x.
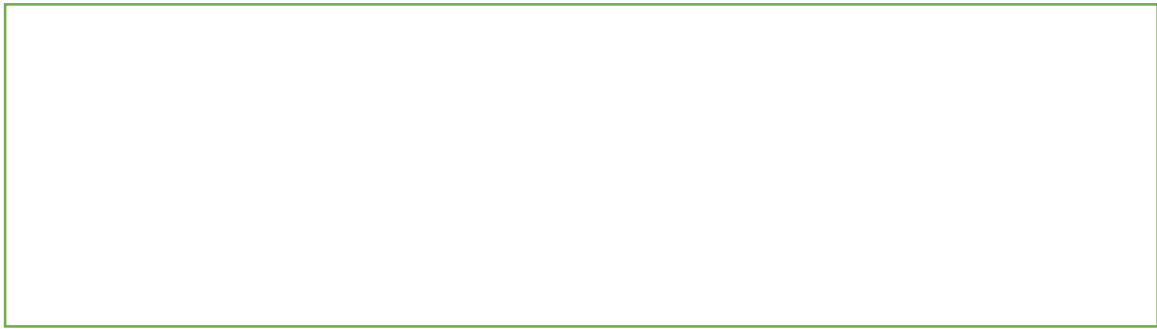Suppose we were to use a linear model $y = mx + c$.

a) Determine m and c (you can assume the line goes through $(0,70)$ and $(250,20)$.
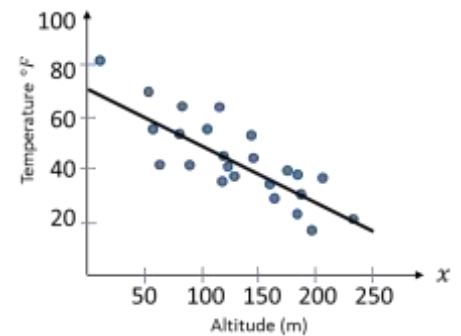
b) Interpret the meaning of m and c in this context

c) Predict at what altitude the temperature reaches $0°F$

Evaluating a Model

Example:

The current population of Bickerstonia is 26000. This year (2017) the population increased by 150. Matt decides to model the population P based on the years t after 2017 by the linear model:

$$P = mt + c$$

Why might this not be a suitable model?