MechYr2 Chapter 6 :: Projectiles

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Last modified: 1st August 2018

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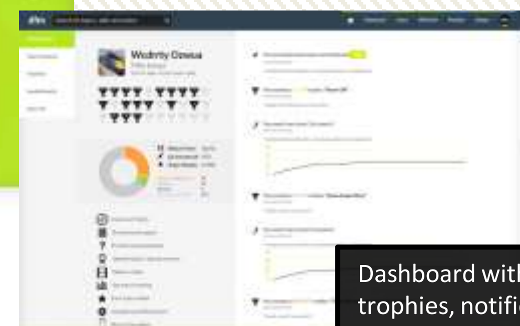

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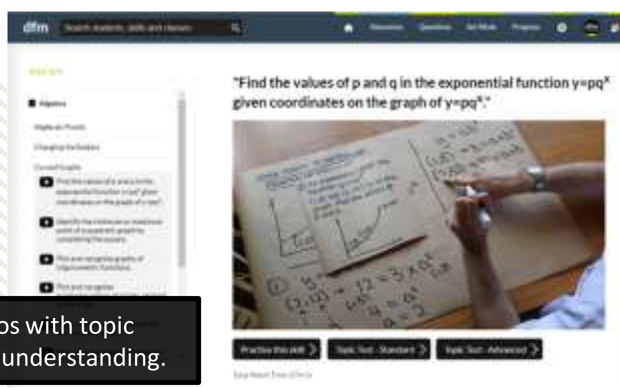
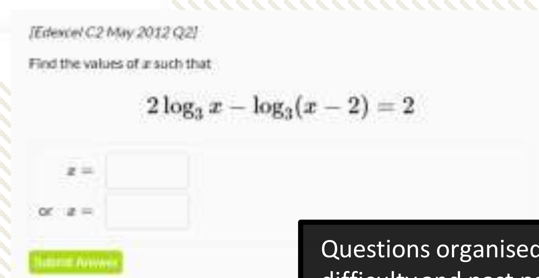


Teaching videos with topic tests to check understanding.



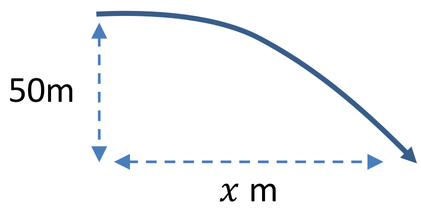
Questions organised by topic, difficulty and past paper.

Overview

In Mechanics Year 1 we already encountered problems of vertical motion of objects when projected vertically. We used "suvat" equations where the acceleration was g ms^{-2}.
In this chapter we allow the object to be **projected sideways**!

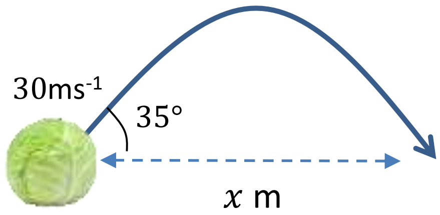
1:: Horizontally projected

"A particle is projected horizontally at 20 ms^{-1}, at a distance 50m above the ground. How far along the ground does it travel?"



2:: Projection at any angle

"A cabbage is projected from ground level at 30 ms^{-1} at an angle of 35°. How far away is the cabbage when it hits the ground?"



Acceleration in each direction.

The key is separately considering the motion in the vertical and horizontal directions:

In **vertical** direction, acceleration downwards is ?

Use suvat equations as before.

In **horizontal** direction, acceleration is ?

Constant velocity, so can use bog standard $speed = \frac{distance}{time}$

A ball is thrown horizontally with speed 20ms^{-1}, from the top of a building, which is 30m high. Find:

a) The time the ball takes to reach the ground.

b) The distance between the bottom of the building and the point where the ball hits the ground.

? Diagram

a ?

b ?

Further Example

A particle is projected horizontally with a velocity of $39.2ms^{-1}$. Find the horizontal and vertical components of the velocity of the particle 3s after projection. Find also the speed and direction of the motion of the particle.

? Diagram

? Working

? Working

? Working

Exercise 6A

Pearson Stats/Mechanics Year 2

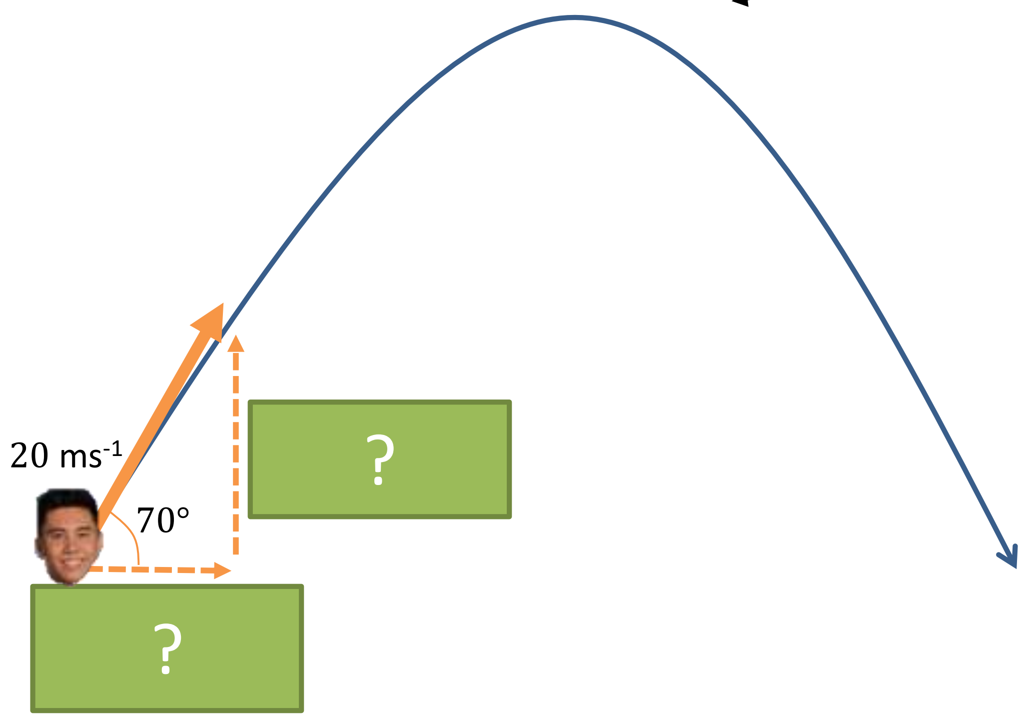
Pages 110-111

Components of velocity

Just as **we split forces into its horizontal and vertical components**, in order to consider forces in the horizontal and vertical directions respectively, we can do **exactly the same with velocity**!

When the object is at its highest point:

?



Click for Tomanimation

We know that the scalar form of velocity is **speed**, and thus we just find the **magnitude** of the velocity vector:

$$\begin{pmatrix} 12 \\ 5 \end{pmatrix} ms^{-1}$$

$\Rightarrow$?

Simple Example *(Textbook Exercise 6B Q4)*

A particle is projected from the top of a building with initial velocity of 28ms^{-1} at an angle *q* below the horizontal, where $\tan\theta = \frac{7}{24}$.
a) Find the horizontal and vertical components of the initial velocity
b) Express the initial velocity as a vector in terms of ***i*** and ***j***.

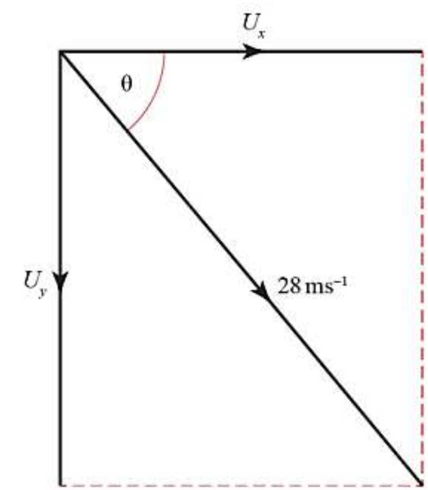
a

? Working

? Working

b

? Working



$$\tan\theta = \frac{7}{24}$$

$\cos\theta =$?

$\sin\theta =$?

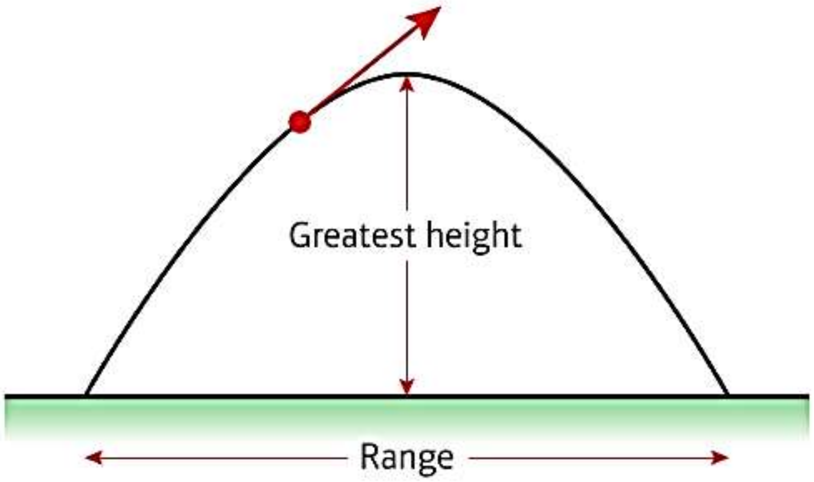
Exercise 6B

Pearson Stats/Mechanics Year 2

Pages 112-113

Projection at Any Angle

You can solve problems involving particles projected at any angle by resolving the initial velocity into horizontal and vertical components.



The distance from the point from which the particle was projected to the point where it strikes the horizontal plane is called the **range**.

The time the particle takes to move from its point of projection to the point where it strikes the horizontal plane is called the **time of flight** of the projectile.

- **A projectile reaches its point of greatest height when the vertical component of its velocity is equal to 0.**

Projection at Any Angle

A particle is projected from a point on a horizontal plane and has an initial velocity of $28\sqrt{3}ms^{-1}$ at an angle of elevation of 60^{o}. Find the greatest height reached by the particle and the time taken to reach this point. Also find the range of the particle.

? Diagram

(It's important you draw one!)

To find the greatest height:

?

?

To find the time taken to reach the greatest height:

?

To find the range, remember that the time of flight is twice the time to reach the greatest height:

?

Example

A golfer hits a ball with a velocity of $52ms^{-1}$, at an angle α above the horizontal where $\tan\alpha = \frac{5}{12}$.

a) Set up a mathematical model, stating any assumptions made
b) Determine the time for which the ball is at least 15m above the ground (take $g = 10ms^{-2}$)

The key is to find the two times at which the particle is 15m above ground. The time above 15m will then be the difference between these times.

? Diagram

(It's important you draw one!)

a

?

b

?

?

$$\tan\alpha = \frac{5}{12} \Rightarrow \sin\alpha = \ ? \ \ and \cos\alpha = \ ?$$

Test Your Understanding

Edexcel M2(Old) May 2012 Q7

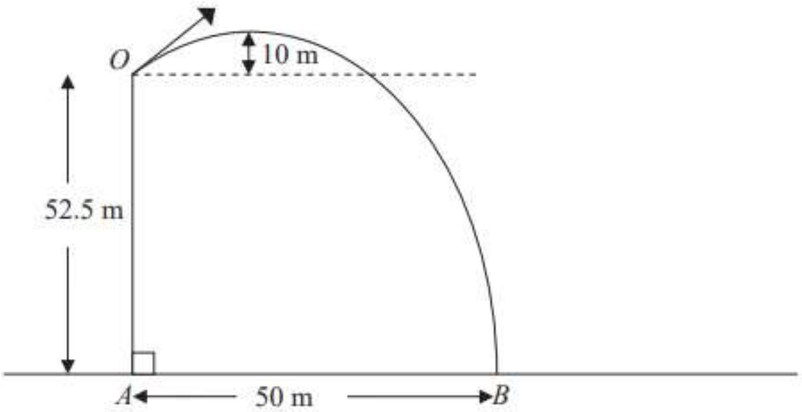


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.
(3)

(b) Find the speed of projection.
(9)

(c) Find the time after projection when the stone is moving parallel to OB.
(5)

a

?

b

?

c

?

Exercise 6C

Pearson Stats/Mechanics Year 2
Pages 117-120

A ball is projected from ground level at an angle of θ. Prove that when the ball hits the ground, the distance the ball has travelled along the ground is maximised when $\theta = 45°$.
(Year 2 differentiation knowledge required)

?

Projectile Motion Formulae

There's nothing new here, but you may be asked to prove more general results regarding projectile motion.

[Textbook] A particle is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal and moves freely under gravity until it hits the plane at point B. Given that that acceleration due to gravity is g, find expressions for:

(a) the time of flight, T

(b) the range, R, on the horizontal plane.

a

?

b

?

Projectile Formulae

[Textbook] A particle is projected from a point with speed U at an angle of elevation α and moves freely under gravity. When the particle has moved a horizontal distance x, its height above the point of projection is y.

(a) Show that $y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$

A particle is projected from a point O on a horizontal plane, with speed 28 ms^{-1} at an angle of elevation α. The particle passes through a point B, which is at a horizontal distance of 32m from O and at a height of 8m above the plane.

(b) Find the two possible values of α, giving your answers to the nearest degree.

a ?

?

?

Don't be intimidated by the lack of numerical values. Just do what you'd usually do and resolve both vertically and horizontally!

Projectile Formulae

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(b) Find the two possible values of α, giving your answers to the nearest degree.

b

?

?

General Results

Exam Note: You may be asked to derive these. But don't attempt to memorise them or actually use them to solve exam problems – instead use the techniques used earlier in the chapter.

For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \frac{2U\sin\alpha}{g}$
- Time to reach greatest height $= \frac{U\sin\alpha}{g}$
- Range on horizontal plane $= \frac{U^2\sin 2\alpha}{g}$
- Equation of trajectory: $y = x\tan\alpha - \frac{gx^2}{2U^2}(1+\tan^2\alpha)$
  where y is vertical height of particle and x horizontal distance.

Exercise 6D

Pearson Stats/Mechanics Year 2
Pages 123-125