P2 Chapter 6 :: Trigonometry

Chapter Overview

This chapter is very similar to the trigonometry chapters in Year 1. The only difference is that new trig functions: $sec, cosec$ and cot, are introduced.

1:: Understanding $sec, cosec, tan$ and draw their graphs.

"Draw a graph of $y = cosec\ x$ for $0 \leq x < 2\pi$."

2:: 'Solvey' questions.

"Solve, for $0 \leq x < 2\pi$, the equation

$$2cosec^2 x + \cot x = 5$$

giving your solutions to 3sf."

3:: 'Provey' questions.

"Prove that

$$\sec x - \cos x \equiv \sin x \tan x$$"

4:: Inverse trig functions and their domains/ranges.

"Show that, when θ is small,
$\sin 5\theta + \tan 2\theta - cos2\theta \approx 2\theta^2 + 7\theta - 1$."

Specification

5.4	Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.	Angles measured in both degrees and radians.

5.5	**Understand and use** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ **Understand and use** $\sin^2\theta + \cos^2\theta = 1$ $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$	**These identities may be used to solve trigonometric equations** and angles may be in degrees or radians. **They may also be used to prove further identities.**
5.6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, and $\tan(A \pm B)$, understand geometrical proofs of these formulae.	To include application to half angles. Knowledge of the $\tan(\frac{1}{2}\theta)$ formulae will *not* be required.
	Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$	Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval.
5.7	**Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.**	**Students should be able to solve equations such as** $\sin(x + 70°) = 0.5$ **for** $0 < x < 360°$, $3 + 5\cos 2x = 1$ **for** $-180° < x < 180°$ $6\cos^2 x + \sin x - 5 = 0,\ 0 \leqslant x < 360°$ These may be in degrees or radians and this will be specified in the question.
5.8	Construct proofs involving trigonometric functions and identities.	Students need to prove identities such as $\cos x \cos 2x + \sin x \sin 2x \equiv \cos x$.
5.9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	Problems could involve (for example) wave motion, the height of a point on a vertical circular wheel, or the hours of sunlight throughout the year. Angles may be measured in degrees or in radians.

Reciprocal Trigonometric Functions

$$\sec(x) = \frac{1}{\cos(x)}$$

Short for "**secant**"

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

Short for "**cosecant**"

$$\cot(x) = \frac{1}{\tan(x)} \; or \; \frac{\cos(x)}{\sin(x)}$$

Short for "**cotangent**"

Calculations

You have a calculator in A Level exams, but won't however in STEP, etc. It's good however to know how to calculate certain values yourself if needed.

$\cot\frac{\pi}{4} =$ $\boxed{}$

$\sec\frac{\pi}{4} =$ $\boxed{}$

$\operatorname{cosec}\frac{\pi}{3} =$ $\boxed{}$

$\cot\frac{\pi}{6} =$ $\boxed{}$

$\operatorname{cosec}\frac{5\pi}{6} =$ $\boxed{}$

$\cot\frac{\pi}{3} =$ $\boxed{}$

$\sec\frac{\pi}{6} =$ $\boxed{}$

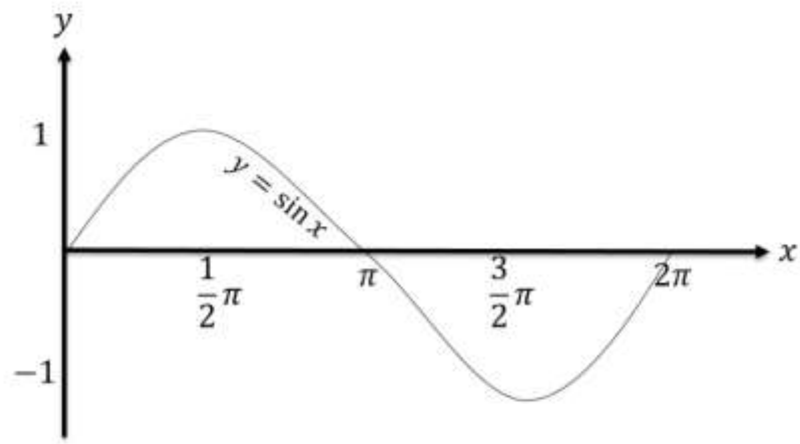
$\operatorname{cosec}\frac{\pi}{2} =$ $\boxed{}$

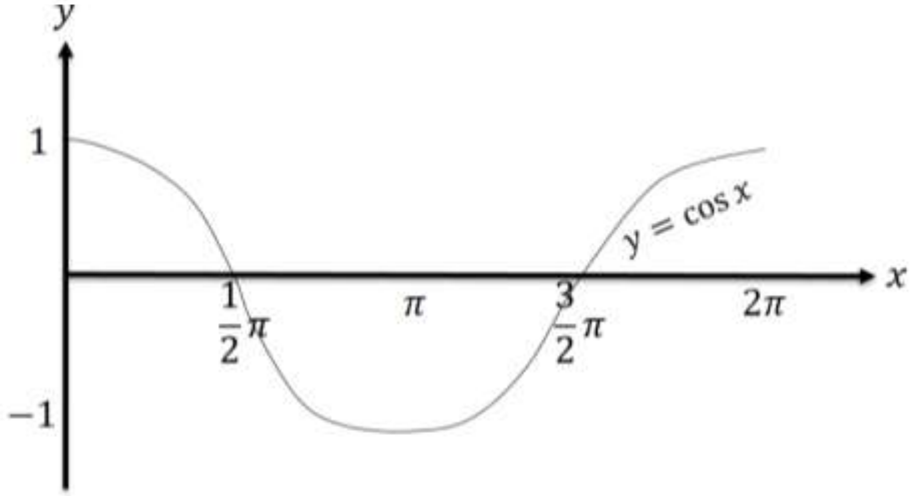
$\sec\frac{5\pi}{3} =$ $\boxed{}$

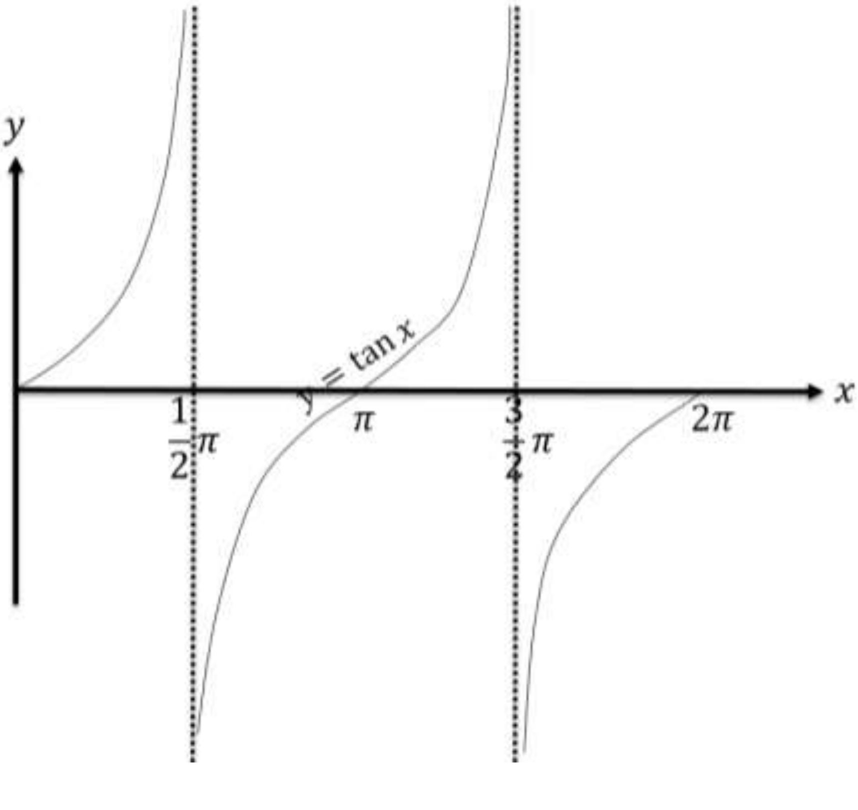
Exercise 6A Page 144

Sketches

To draw a graph of $y = cosec\ x$, start with a graph of $y = \sin x$, then consider what happens when we reciprocate each y value.







Example

[Textbook]
a) Sketch the graph of $y = 4cosec\ x, \quad -\pi \leq x \leq \pi$.
b) On the same axes, sketch the line $y = x$.
c) State the number of solutions to the equation $4cosec\ x - x = 0, \ -\pi \leq x \leq \pi$

Test Your Understanding

Sketch $y = -1 + \sec 2x$ in the interval $0 \leq x < 360°$.

Exercise 6B Page 148

Using $sec, cosec, cot$

Questions in the exam usually come in two flavours: (a) 'provey' questions requiring to prove some identity and (b) 'solvey' questions.

[Textbook]
(a) Simplify $\sin\theta \cot\theta \sec\theta$
(b) Simplify $\sin\theta \cos\theta \ (\sec\theta + cosec\ \theta)$
(c) Prove that $\frac{\cot\theta\ cosec\ \theta}{\sec^2\theta + cosec^2\theta} \equiv \cos^3\theta$

Tip 1: Get everything in terms of sin and cos first (using $\cot x = \frac{\cos x}{\sin x}$ rather than $\cot x = \frac{1}{\tan x}$)

Tip 2: Whenever you have algebraic fractions being added/subtracted, whether $\frac{a}{b} + \frac{c}{d}$ or $\frac{a}{b} + c$, combine them into one (as we can typically then use $\sin^2 x + \cos^2 x = 1$)

Test Your Understanding

1 $\sec x - \cos x \equiv \sin x \tan x$

2 $(1 + \cos x)(cosec\ x - \cot x) \equiv \sin x$

Solvey Questions

[Textbook] Solve the following equations in the interval $0 \leq \theta \leq 360°$:
a) $\sec\theta = -2.5$
b) $\cot 2\theta = 0.6$

a

b

Solve $\cot\theta = 0$ in the interval $0 \leq \theta \leq 2\pi$.

Test Your Understanding

Solve in the interval $0 \leq \theta < 360°$:

$$cosec\ 3\theta = 2$$

Exercise 6C Page 152

New Identities

$$\sin^2 x + \cos^2 x = 1$$

There are just two new identities you need to know:

Dividing by $\cos^2 x$:

Dividing by $\sin^2 x$:

"Prove that $1 + \tan^2 x \equiv \sec^2 x$."

$$\sin^2 x + \cos^2 x \equiv 1$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} \equiv \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 \equiv \sec^2 x$$

Examples

[Textbook] Prove that $cosec^4\,\theta - \cot^4\theta = \frac{1+\cos^2\theta}{1-\cos^2\theta}$

Solve the equation $4\,cosec^2\theta - 9 = \cot\theta$ in the interval $0 \leq \theta \leq 360°$

This is just like in AS; if you had say a mixture of $\sin\theta$, $\sin^2\theta$, $\cos^2\theta$: you'd change the $\cos^2\theta$ to $1 - \sin^2\theta$ in order to get a quadratic in terms of sin.

Test Your Understanding

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6. (ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

$$3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$$

Q

Solve, for $0 \leq x < 2\pi$, the equation

$$2cosec^2 x + \cot x = 5$$

giving your solutions to 3sf.

Inverse Trig Functions

$$If\ sinx = \frac{1}{2}\ then\ x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$We\ also\ call\ this\ \arcsin(\frac{1}{2})\ so\ we\ say\ x = \arcsin(\frac{1}{2})$$

$$\boldsymbol{The\ inverse\ trig\ functions\ are\ known\ as}$$

$$\boldsymbol{y = \arcsin x\,, y = \arccos x, y = \arctan x}$$

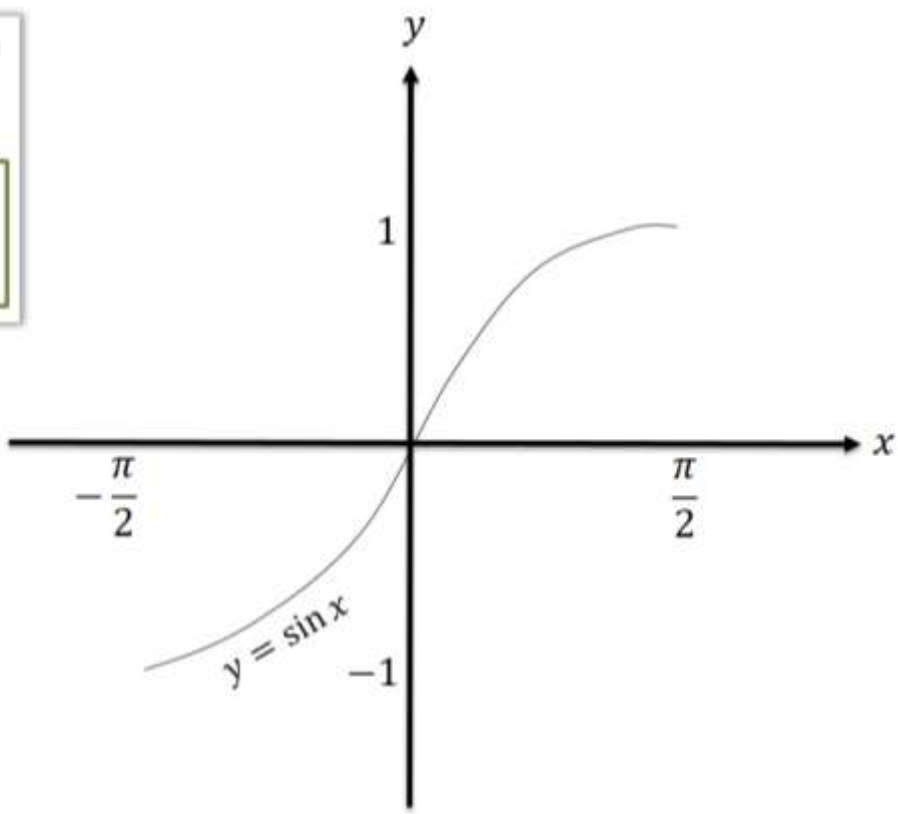
They are inverse functions, hence

- They only exist for a one to one function
- They map from the range of the original function back to its original domain
- The graphs are reflections of the original in the line y = x.

Inverse Trig Functions

You need to know how to sketch $y = \arcsin x\,, y = \arccos x, y = \arctan x$.
(Yes, you could be asked in an exam!)

We have to restrict the domain of $\sin x$ to $-\frac{\pi}{2} \leq x < \frac{\pi}{2}$ before we can find the inverse. Why?



Inverse Trig Functions

$y = \arccos x$

$y = \arctan x$

Evaluating inverse trig functions

[Textbook] Work out, in radians, the values of:

a) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

b) $\arccos(-1)$

c) $\arctan\left(\sqrt{3}\right)$

You can simply use the $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$ buttons on your calculator.
If you don't have a calculator, just use the sin, cos, tan graphs backwards.

One Final Problem…

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8. (ii) Given that

$$y = \arccos x, \quad -1 \le x \le 1 \quad \text{and} \quad 0 \le y \le \pi,$$

(*a*) express $\arcsin x$ in terms of y.

(2)

(*b*) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)