MechYr2 Chapter 5 :: Friction

jfrost@tiffin.kingston.sch.uk

www.drfrostmaths.com

@DrFrostMaths

Last modified: 31st July 2018

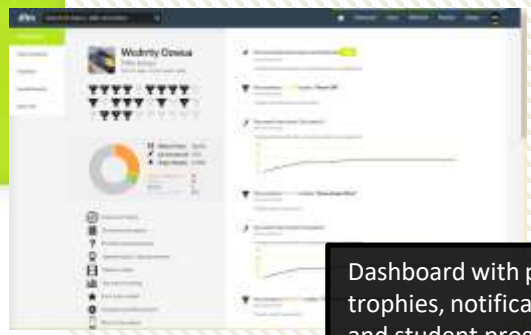
www.drfrostmaths.com

Everything is **completely free**.
Why not register?

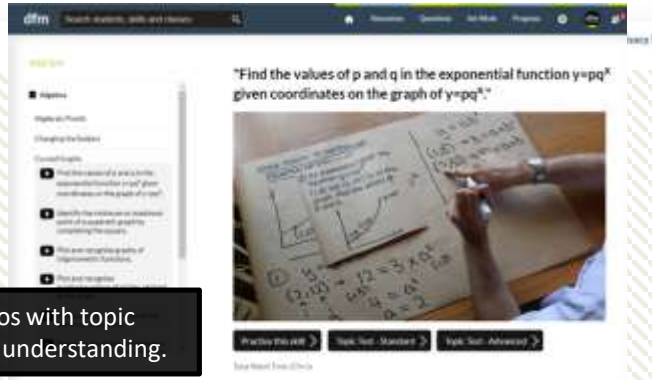
Register now to interactively practise questions on this topic, including past paper questions and extension questions (including MAT + UKMT). Teachers: you can create student accounts (or students can register themselves), to set work, monitor progress and even create worksheets.

With questions by:

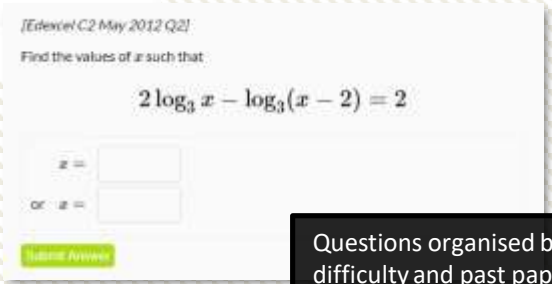
edexcel OCR AQA UKMT



Dashboard with points, trophies, notifications and student progress.



Teaching videos with topic tests to check understanding.



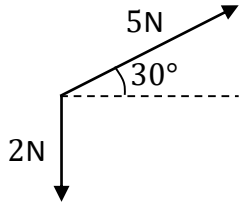
Questions organised by topic, difficulty and past paper.

Overview

In Year 1 any frictional forces were stated. In this chapter, we will be able to **calculate the frictional force** using the normal reaction force acting on the object.

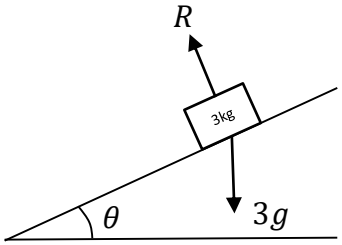
1:: Resolving components

"Determine the magnitude and direction of the resultant force."



2:: Inclined Planes

"A block of mass 3kg is placed on a smooth slope with angle of inclination θ where $\tan\theta = \frac{3}{4}$. Determine the acceleration of the block down the slope."

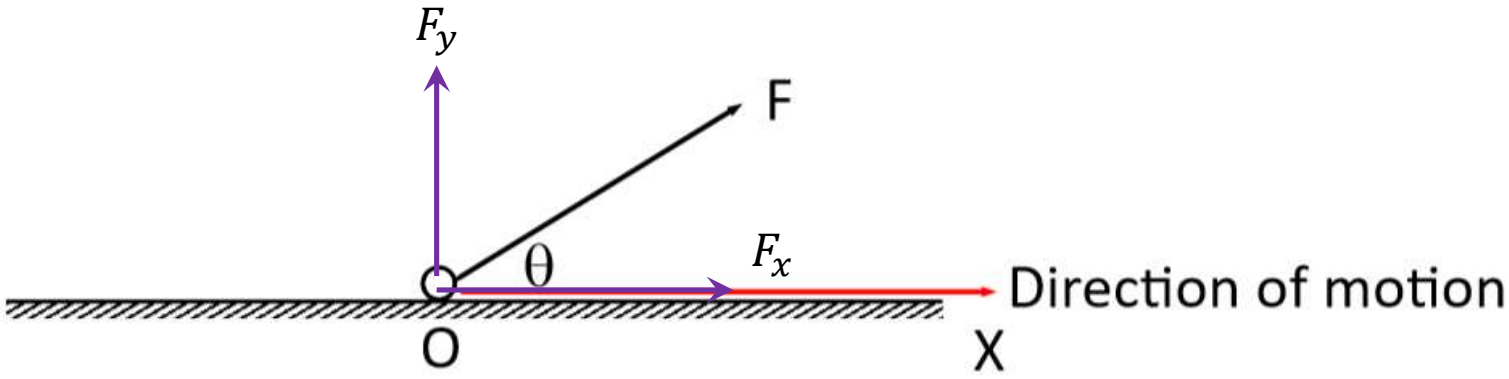


3:: $F \leq \mu R$

Understand that the maximum friction is μR, where μ is the coefficient of friction of the surface, and R is the normal reaction force of the surface on the object. Use to solve inclined plane problems when the surface is rough.

Resolving Forces

If a force is applied at an angle to the direction of motion you can resolve it to find the component of the force that acts in the direction of motion.



The **component** of a force, F, in any given direction is the measure of the effect of the force in that direction. Click to see components

In the diagram above, the magnitude of the force in the horizontal and vertical direction can be calculated using trigonometry:

Horizontal component:

?

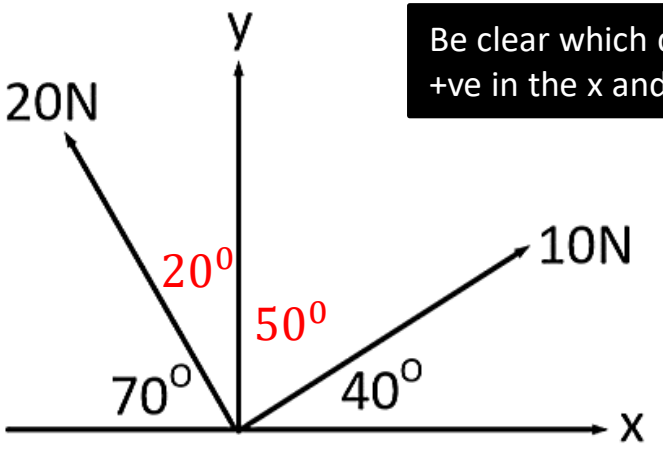
Vertical component:

?

Rule of thumb:
Pull a force THROUGH an angle $\Longrightarrow \cos\theta$
Pull a force AWAY from an angle $\Longrightarrow \sin\theta$

Example

Find the sum of the components of the given forces in the direction of:
a) the x-direction
b) the y-direction



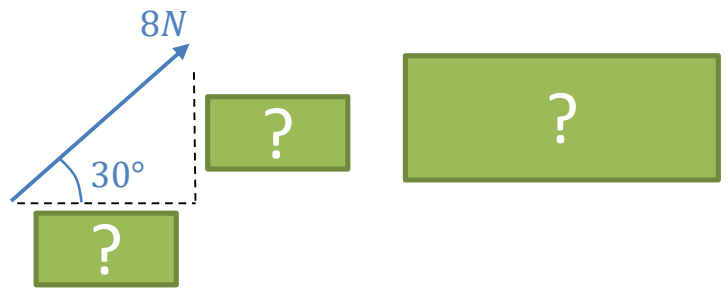
Be clear which directions are +ve in the x and y direction

?

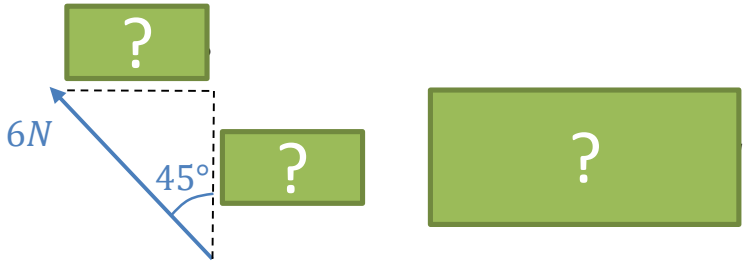
Quickfire Questions

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.

1



2

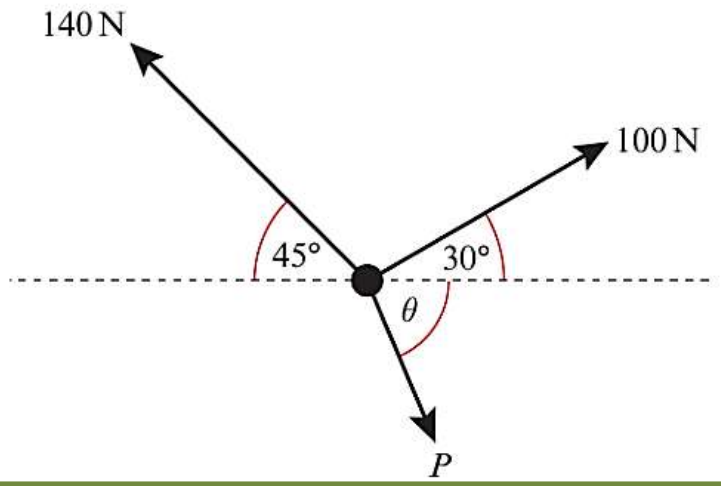


3

Example – Using a Triangle of Forces in Equilibrium

We can also draw a **triangle of forces** to solve problems for particles in equilibrium.

Three forces act on a particle as shown.
Given that the particle is in equilibrium, calculate the magnitude of P.



?

?Diagram

Applied Example

A force *P* is applied to a box of mass 5 kg, causing the box to accelerate at 2 m s^{-2} along a smooth, horizontal plane. Given that the force causing the acceleration is applied at 30° to the plane, work out the value of *P*.

? Diagram

Recall that $F = ma$, but the force F must be in the **same direction as the acceleration**.

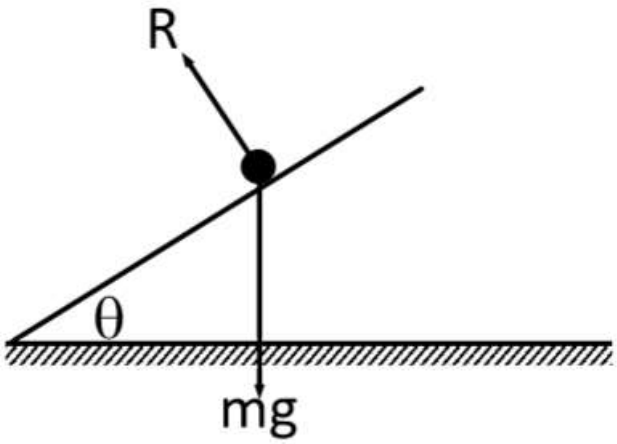
?

Exercise 5A

Pearson Stats/Mechanics Year 2
Pages 94-96

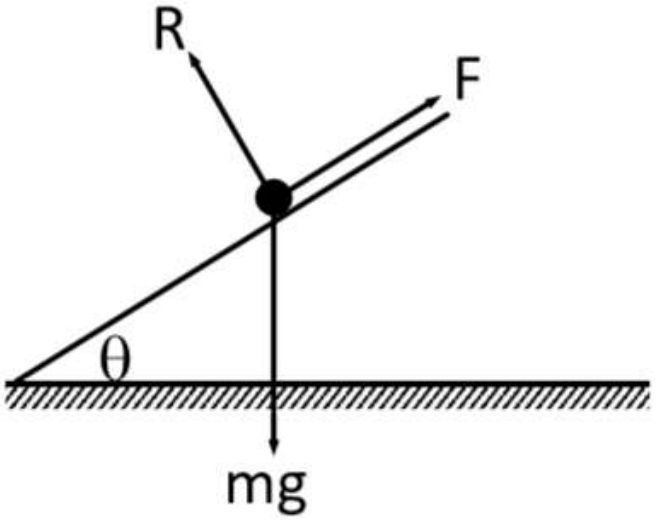
Inclined Planes

For problems involving inclined planes we usually resolve forces parallel and perpendicular to the plane.



Smooth Plane

If there is no friction, the particle will slide down the slope.



Rough Plane

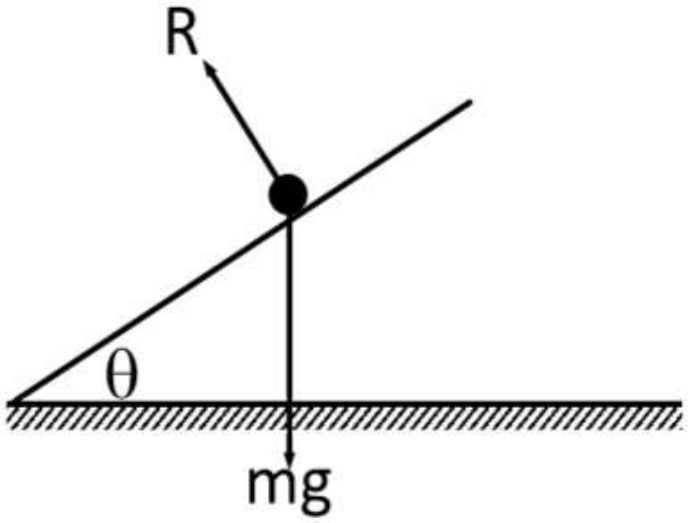
If rough enough, the force of friction *might* be enough to prevent the particle from moving.

Inclined Planes

Remember:

- Weight always acts vertically downwards
- The **normal reaction** always acts perpendicular to the plane
- Friction always opposes the direction of motion

We have to resolve the weight into its components parallel and perpendicular to the plane.



Example

A block of mass 25kg slides down a smooth slope angled at 20° to the horizontal.

a) Draw a force diagram to show all the forces acting on the block

b) Calculate the magnitude of the normal reaction of the slope on the block.

c) Find the acceleration.

? Diagram

? Working

? Working

? Working

? Working

Test your understanding

A block of mass 10kg slides down a smooth slope angled at 15° to the horizontal.

a) Draw a force diagram to show all the forces acting on the block

b) Calculate the magnitude of the normal reaction of the slope on the block.

c) Find the acceleration.

? Diagram

? Working

? Working

? Working

? Working

Inclined Plane with an additional force

A small parcel of mass 2 kg is held in equilibrium on a rough plane by a horizontal force of magnitude 20 N, acting in a vertical plane through a line of greatest slope. The plane is inclined at an angle of 20° to the horizontal. The parcel is modelled as a particle. The parcel is on the point of moving **up** the slope.

a Draw a force diagram to show all the forces acting on the parcel.

b Calculate the magnitude of the normal reaction of the slope on the parcel.

It may help to draw the

? Diagram

? Working

Example

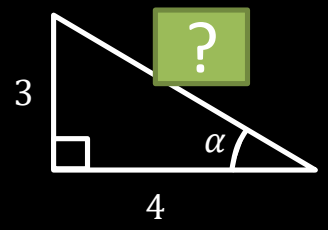
[Textbook Ex 5B] A particle of mass 0.5kg is held at rest on a smooth slope that is inclined at an angle of α to the horizontal. The particle is released. Given that $\tan \propto = \frac{3}{4}$, calculate:

a) The normal reaction between the particle and the plane
b) The acceleration of the particle

? Diagram

? Working

Help: Don't find α explicitly. We can find $\cos\alpha$ and $\sin\alpha$ by forming a suitable triangle such that $\tan\alpha$ would be $\frac{3}{4}$:



$\cos\alpha =$?

$\sin\alpha =$?

Test Your Understanding

[Textbook] A particle of mass m is pushed up a smooth slope, inclined at $30°$ by a force of magnitude $5g$ N acting at angle of $60°$ to the slope, causing the particle to accelerate up the slope at 0.5 ms^{-2}. Show that the mass of the particle is $\left(\frac{5g}{1+g}\right)$ kg

? Diagram

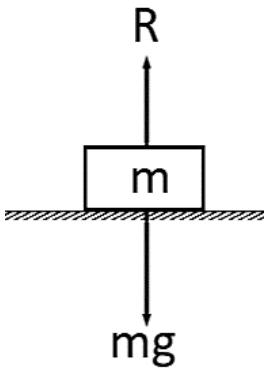
? Working

Exercise 5B

Pearson Stats/Mechanics Year 2
Pages 98-99

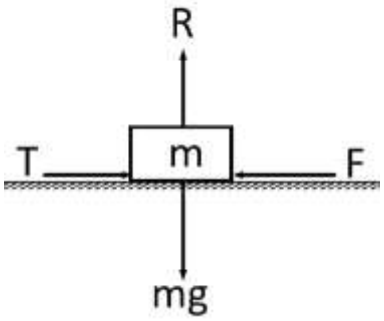
Friction

Scenario 1: A block is on a horizontal rough surface with no forces (other than gravity) acting on it.



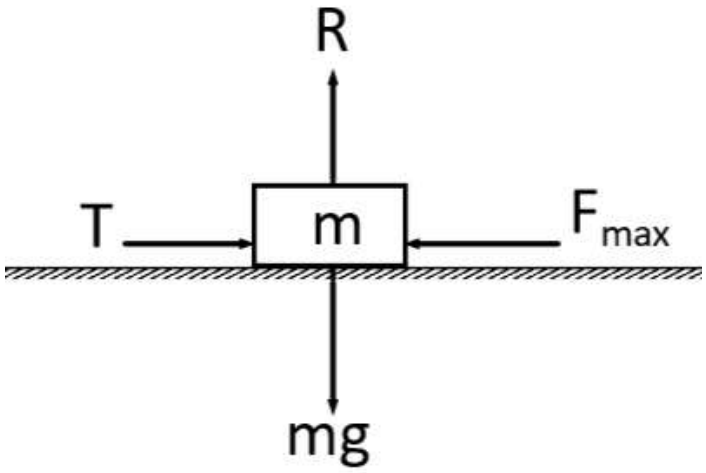
?

Scenario 2: A horizontal force is applied but it is not enough to move the block.



?

Scenario 3: T has reached or exceeded F_{max}, the maximum or limiting value for the friction.



?

Friction

This ‘maximum friction’ depends on two things:

- ?
- ?

✎ The maximum friction between two surfaces:

$$F_{max} = \mu R$$

where μ is the coefficient of friction and R is the normal reaction between two surfaces.

Example μ: (source physlink.com)

Materials	Coeff. of Static Friction μ_s
Steel on Steel	0.74
Aluminum on Steel	0.61
Copper on Steel	0.53
Rubber on Concrete	1.0
Wood on Wood	0.25-0.5
Glass on Glass	0.94
Waxed wood on Wet snow	0.14
Waxed wood on Dry snow	-
Metal on Metal (lubricated)	
Ice on Ice	0.1
Teflon on Teflon	0.04
Synovial joints in humans	0.01

Example

A block of mass 5kg rests on a rough horizontal plane. The coefficient of friction between the block and the plane is 0.6.
Calculate the frictional force acting on the block when a horizontal force, P, is applied to the block and the magnitude of P is:
a) 12N
b) 29.4N
c) 36N
Also calculate the magnitude of any acceleration that may occur.

? Diagram

a

? Working

? Working

b

? Working

c

? Working

Tip: Resolve forces vertically, as we get an expression for R which can then be used in μR when we resolve parallel to the plane.

Inclined Plane with Friction Example

A particle is held at rest on a rough plane, which is inclined to the horizontal at an angle α, where $\tan \alpha = 0.75$. The coefficient of friction between the particle and the plane is 0.5. The particle is released and slides down the plane. Find:

a) The acceleration of the particle

b) The distance it slides in the first two seconds

? Diagram

a

? Working

? Working

? Working

b

? Working

Test Your Understanding

1. A particle *P* of mass 3 kg is held at rest in equilibrium on a rough plane that is inclined to the horizontal at an angle of 30° by a constant force of magnitude xN acting up the plane.
The coefficient of friction between *P* and the plane is *0.5.* Find the magnitude of x if:

a) The particle is on the point of slipping up the plane
b) The particle is on the point of slipping down the plane

?Diagram

?

?

Test Your Understanding

2. A particle P of mass 3 kg is held at rest in equilibrium on a rough plane that is inclined to the horizontal at an angle of 30° by a constant pushing force of magnitude 30 N. The direction of the force is inclined to the plane at an angle of 40°, and its line of action lies in the vertical plane containing P and a line of greatest slope of the plane.
The coefficient of friction between P and the plane is μ.

Given that P is on the point of sliding **up** the plane, find the value of μ.

?Diagram

?

Exercise 5C

Pearson Stats/Mechanics Year 2
Pages 103-104

Additional question:

1 A block lies on a rough plane at an incline of θ. The coefficient of friction between the block and plane is μ. If the block is on the verge of sliding down the plane, prove that $\mu = \tan\theta$.

?

Side Note: Since $\tan 45 = 1$, the implication is that it's very much possible to have values of μ greater than 1, i.e. if we have to raise the angle of the plane beyond $45°$ before the block starts sliding.