Chapter 8 - Mechanics

Modelling in Mechanics

Chapter Overview

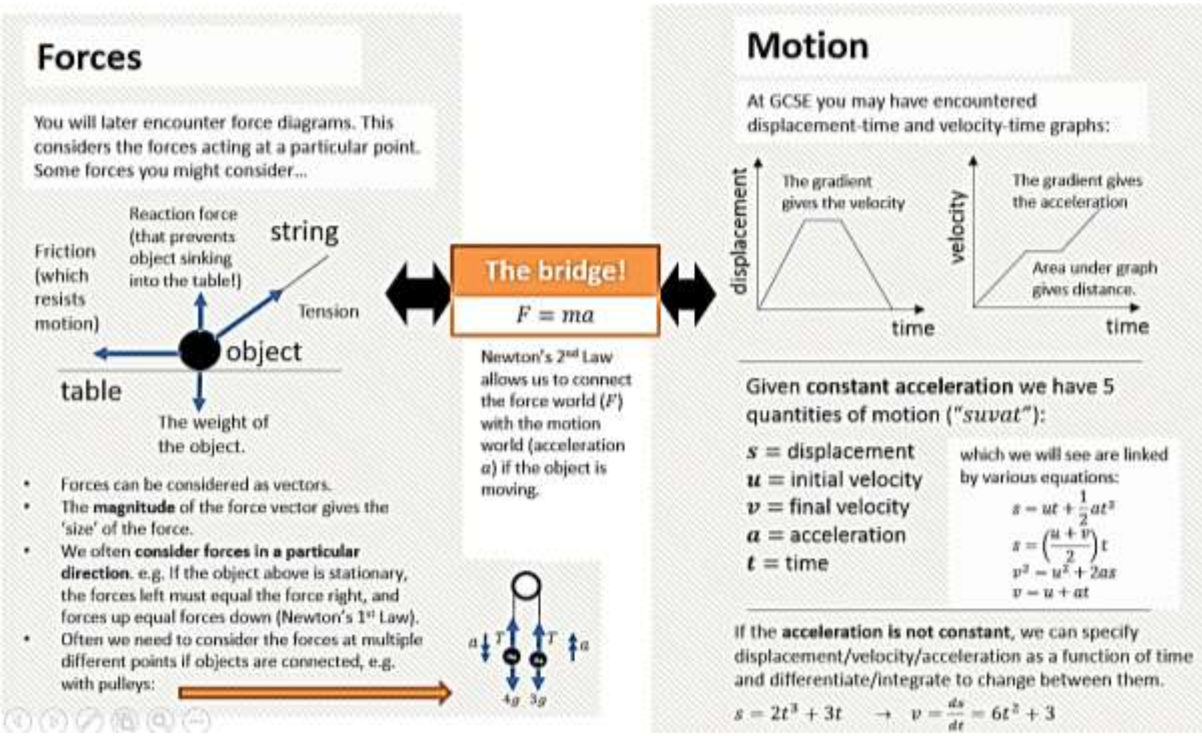
1. Constructing a Model
2. Modelling Assumptions
3. Quantities and Units
4. Working with Vectors

Topics	What students need to learn:		
	Content		Guidance
6 **Quantities and units in mechanics**	6.1	Understand and use fundamental quantities and units in the S.I. system: length, time, mass. Understand and use derived quantities and units: velocity, acceleration, force, weight.	

What is Mechanics?

Broadly speaking, mechanics covers motion, forces and how the two inter-relate with each other.

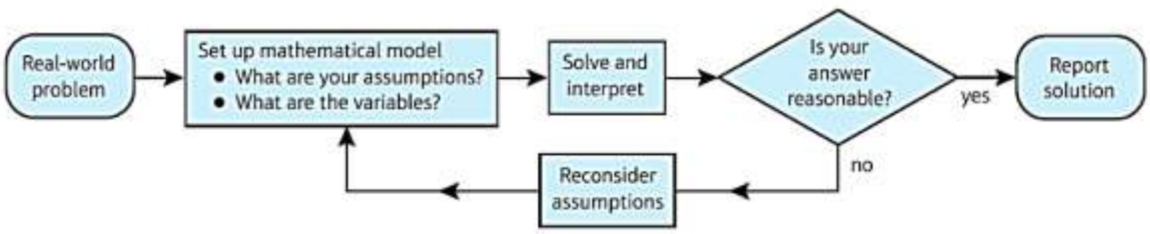
Mechanics, broadly speaking, concerns motion, forces, and how the two interrelate. This chapter just gives you an overview of what you'll be covering in Year 1 and how it all links together.



1. Constructing a Model

Why use a mathematical model?

The solution to a mathematical model needs to be interpreted in the context of the original problem. You may need to refine the model and reconsider your original assumptions.



Example *(Textbook)*

The motion of a basketball as it leaves a player's hand and passes through the net can be modelled using the equation $h = 2 + 1.1x - 0.1x^2$, where h m is the height of the basketball above the ground and x m is the horizontal distance travelled.

a Find the height of the basketball: **i** when it is released **ii** at a horizontal distance of 0.5 m.

b Use the model to predict the height of the basketball when it is at a horizontal distance of 15 m from the player.

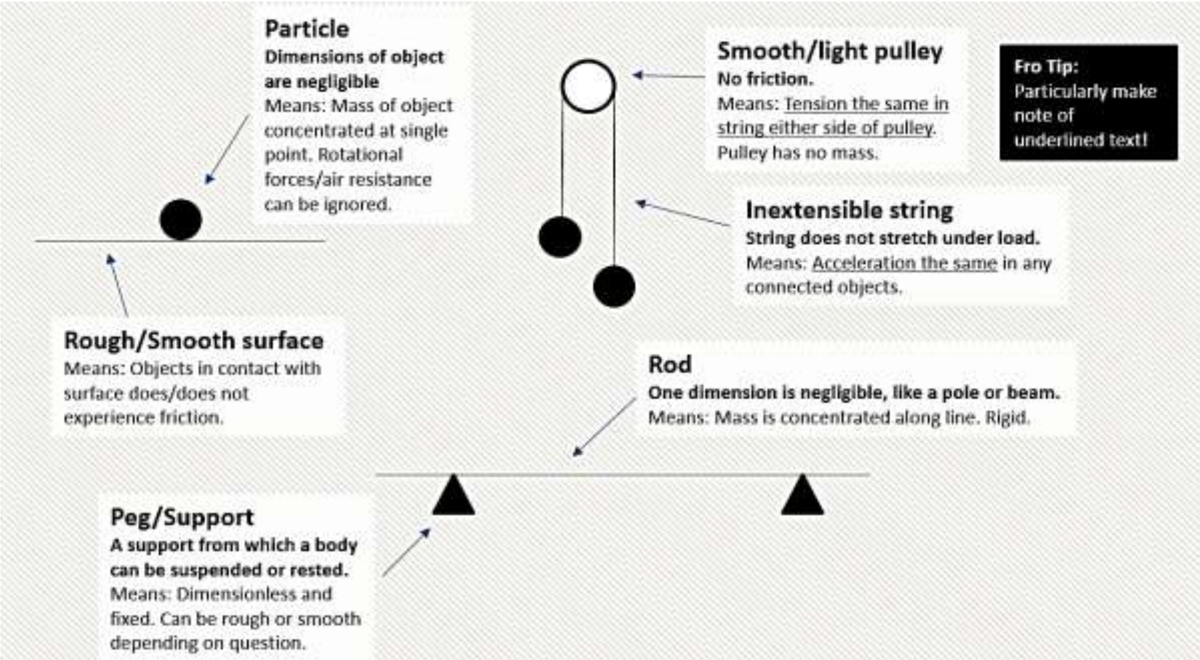
c Comment on the validity of this prediction.

Exercise 8A Page 119

2. Modelling Assumptions

We make modelling assumptions to simplify a problem and solve it using known mathematical techniques. You must be able to understand how these assumptions will affect calculations versus the real-life situation.

A full list of modelling assumptions is on p121 of the textbook. The most common are shown below.



Example *(Exercise 8B Question 2)*

An ice puck is hit and slides across the ice.

State the effect of the following assumptions on any calculations made using this model:

a The ice puck is modelled as a particle. **b** The ice is smooth.

Exercise 8B Page 122

3. Quantities and Units

The SI units are a standard system of units, used internationally ("Système International d'unités"). These are the **base** ones you will use:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/displacement	metre	m
Time	seconds	s

These **derived** units are compound units built from the base units.

Quantity	Unit	Symbol
Speed/velocity	metres per second	$m\,s^{-1}$
Acceleration	metres per second per second	$m\,s^{-2}$
Weight/force	newton	N (= $kg\,m\,s^{-2}$)

Can you convert 2.48×10^5 kmh^{-1} into SI units?

Types of Force and Force Diagrams

You will encounter a variety of forces in mechanics. It is ALWAYS helpful to draw a force diagram and make sure that you have included all forces acting on a body.

- Weight (always vertically downwards)
- Normal Reaction (always perpendicular to the surface of contact)
- Friction (only if the plane is ROUGH, always opposes motion)
- Tension (in a string – PULL force)
- Thrust/compression (e.g. in a rod or engine – PUSH force)
- Resistance (e.g. particle travelling through a liquid, always opposes direction of motion)
- Buoyancy (e.g. boat floating in water, always vertically upwards)

Force diagrams can be found on page 123 of the textbook.

Exercise 8C Page 124

4. Working with Vectors

In Mechanics you will often need to convert to/from the scalar form of a quantity and the vector form.

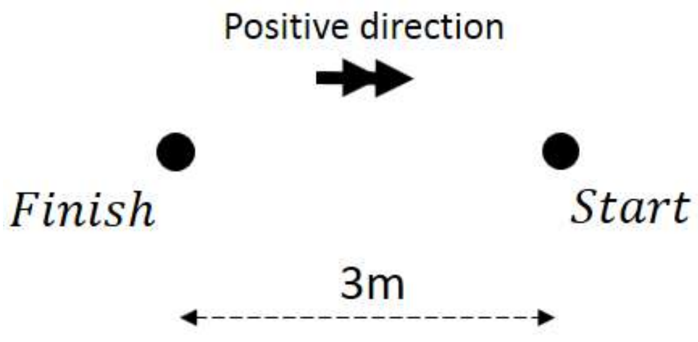
SCALAR:

VECTOR:

Examples of scalars and vectors:

Scalar	**Vector**

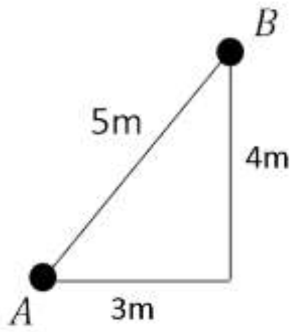
NB: 1-dimensional vectors are still different from scalars. Consider the displacement on a 1-dimensional line in a particular direction. If we'd gone backwards 3 units…



What is the distance travelled?

What is the displacement of the particle?

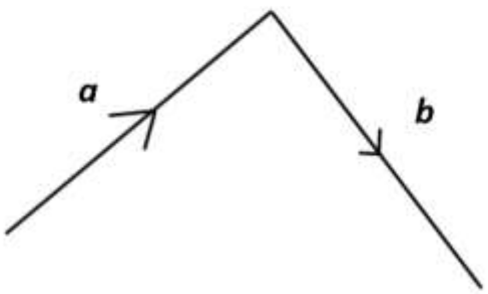
Vector Notation

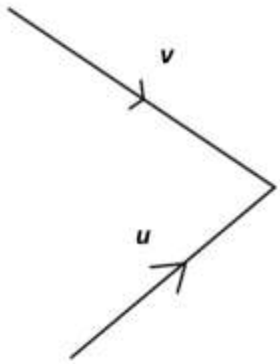


Column Notation

i-j **Notation**
(***i*** and ***j*** are **unit** vectors of length 1)

Adding and Subtracting Vectors





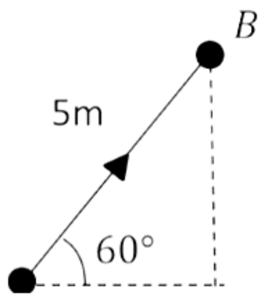
Two vectors are equal if they have the **same magnitude and direction**.

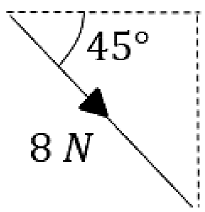
Two vectors are parallel if they have the **same direction but different magnitudes.**

Converting Between Vectors and Scalars

To convert to vector form, just use basic trigonometry to find the x-change and y-change.

Scalar (Distance) **Vector (Displacement)**



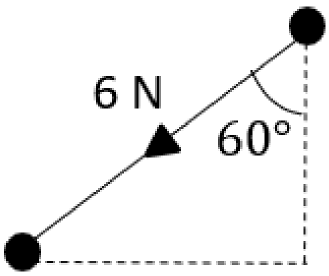
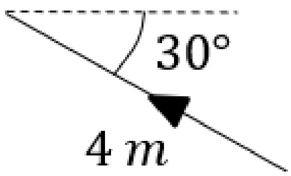


To convert scalar form, just find the **magnitude** of the vector using Pythagoras.

Vector (Velocity) **Scalar (Speed)**

$\begin{pmatrix} 5 \\ -12 \end{pmatrix} ms^{-1}$

Further Examples

Scalar Form	Vector Form
	
	$(6\boldsymbol{i} - 8\boldsymbol{j})\ ms^{-2}$
	

Test Your Understanding

A woman walks from A to B and then from B to C.
Her displacement from A to B is $6\underset{\sim}{i} + 4\underset{\sim}{j}$ m.
Her displacement from B to C is $5\underset{\sim}{i} - 12\underset{\sim}{j}$ m.

a) What is the magnitude of the displacement from A to C?
b) What is the total distance the woman has walked in getting from A to C?

Exercise 8D Page 127
Mixed Exercise 8 Page 128