MechYr1 Chapter 8 :: Introduction to Mechanics

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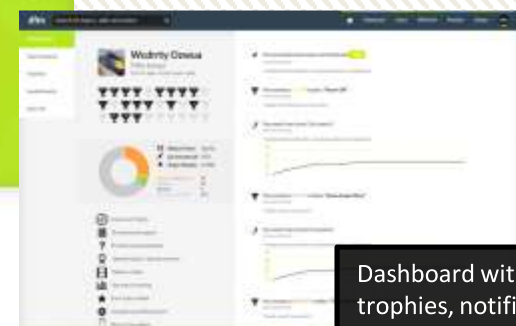

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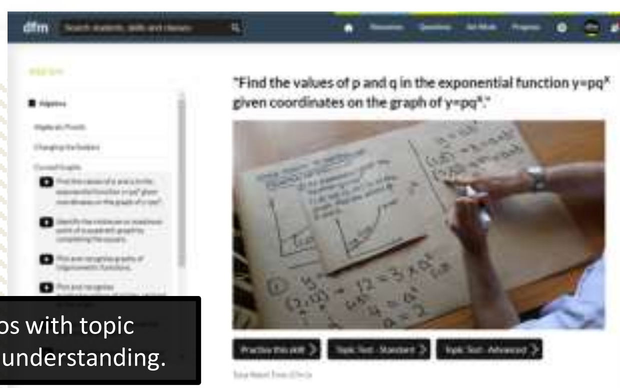
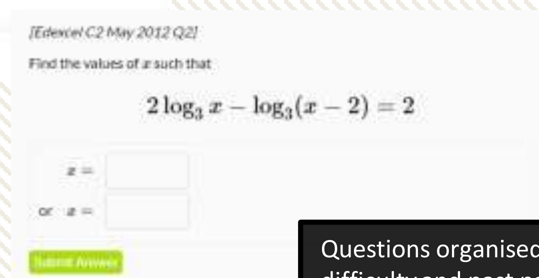


Teaching videos with topic tests to check understanding.

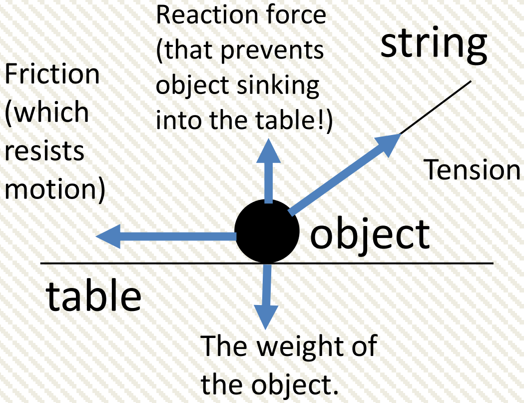


Questions organised by topic, difficulty and past paper.

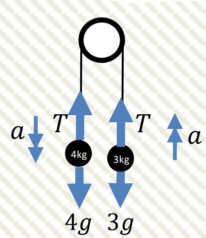
Mechanics, broadly speaking, concerns motion, forces, and how the two interrelate. This chapter just gives you an overview of what you'll be covering in Year 1 and how it all links together.

Forces

You will later encounter force diagrams. This considers the forces acting at a particular point. Some forces you might consider…



- Forces can be considered as vectors.
- The **magnitude** of the force vector gives the 'size' of the force.
- We often **consider forces in a particular direction**. e.g. If the object above is stationary, the forces left must equal the force right, and forces up equal forces down (Newton's 1st Law).
- Often we need to consider the forces at multiple different points if objects are connected, e.g. with pulleys:



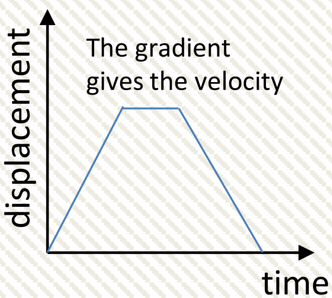
The bridge!

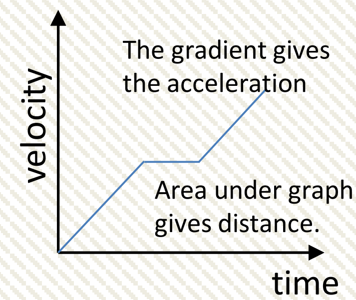
$$F = ma$$

Newton's 2nd Law allows us to connect the force world (F) with the motion world (acceleration a) if the object is moving.

Motion

At GCSE you may have encountered displacement-time and velocity-time graphs:





Given **constant acceleration** we have 5 quantities of motion ("*suvat*"):

$\boldsymbol{s}$ = displacement
$\boldsymbol{u}$ = initial velocity
$\boldsymbol{v}$ = final velocity
$\boldsymbol{a}$ = acceleration
$\boldsymbol{t}$ = time

which we will see are linked by various equations:

$$s = ut + \frac{1}{2}at^2$$
$$s = \left(\frac{u+v}{2}\right)t$$
$$v^2 = u^2 + 2as$$
$$v = u + at$$

If the **acceleration is not constant**, we can specify displacement/velocity/acceleration as a function of time and differentiate/integrate to change between them.

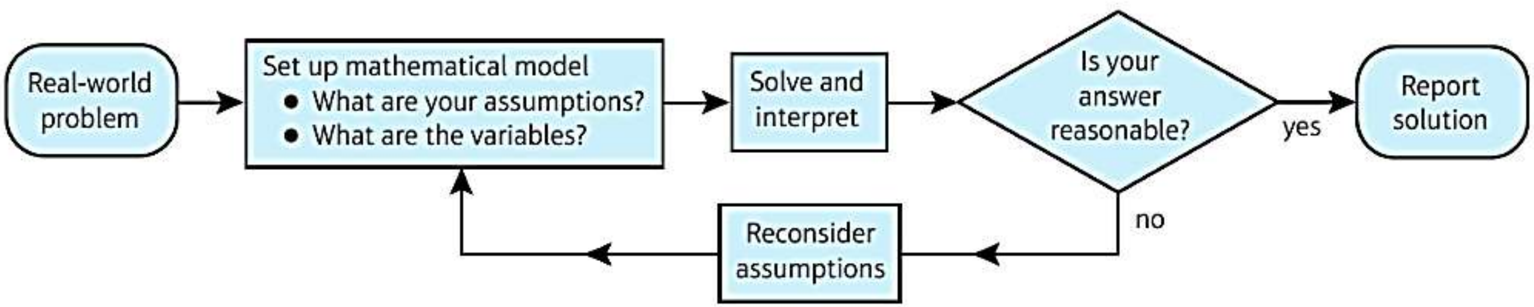
$$s = 2t^3 + 3t \quad \rightarrow \quad v = \frac{ds}{dt} = 6t^2 + 3$$

Constructing a Mathematical Model

Mathematical models are used to simulate real-life situations. Why else might we want to use mathematical modelling?

- We can use equations and graphs to describe the problem which…
- Simplifies the problem by making assumptions in order to solve the problem
- Cheaper than doing it to “real-life” scale!

The solution to a mathematical model needs to be interpreted in the context of the original problem. You may need to refine the model and reconsider your original assumptions.



Example

The motion of a basketball as it leaves a player's hand and passes through the net can be modelled using the equation $h = 2 + 1.1x - 0.1x^2$, where h m is the height of the basketball above the ground and x m is the horizontal distance travelled.

a Find the height of the basketball: **i** when it is released **ii** at a horizontal distance of 0.5 m.

b Use the model to predict the height of the basketball when it is at a horizontal distance of 15 m from the player.

c Comment on the validity of this prediction.

? Suitable Sketch

ai) ?

aii) ?

b ?

c ?

Exercise 8A

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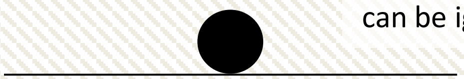
Modelling Assumptions

As with many areas of applied maths, we often have to make various modelling assumptions, to make the maths cleaner or to use well-known mathematical approaches.
Here are common modelling assumptions often made in Mechanics:

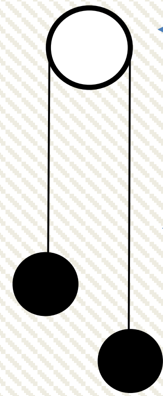


Particle

Dimensions of object are negligible
Means: Mass of object concentrated at single point. Rotational forces/air resistance can be ignored.

Rough/Smooth surface

Means: Objects in contact with surface does/does not experience friction.



Smooth/light pulley

No friction.
Means: Tension the same in string either side of pulley. Pulley has no mass.

Tip: Particularly make note of underlined text!

Inextensible string

String does not stretch under load.
Means: Acceleration the same in any connected objects.

Rod

One dimension is negligible, like a pole or beam.
Means: Mass is concentrated along line. Rigid.

Peg/Support

A support from which a body can be suspended or rested.
Means: Dimensionless and fixed. Can be rough or smooth depending on question.

Example

An ice puck is hit and slides across the ice.

State the effect of the following assumptions on any calculations made using this model:

a The ice puck is modelled as a particle.

b The ice is smooth.

a)

?

b)

?

Exercise 8B

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Quantities and Units

The SI units are a standard system of units, used internationally ("Système International d'unités"). These are the **base** ones you will use:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/displacement	metre	m
Time	seconds	s

These **derived** units are compound units built from the base units.

Quantity	Unit	Symbol
Speed/velocity	metres per second	$m\,s^{-1}$
Acceleration	metres per second per second	$m\,s^{-2}$
Weight/force	newton	N (= $kg\,m\,s^{-2}$)

Can you convert 2.48×10^5 kmh^{-1} into SI units?

Example

Convert $2.48 \times 10^5 kmh^{-1}$ into SI units.

You can use the "conv" button on your calculator to help!

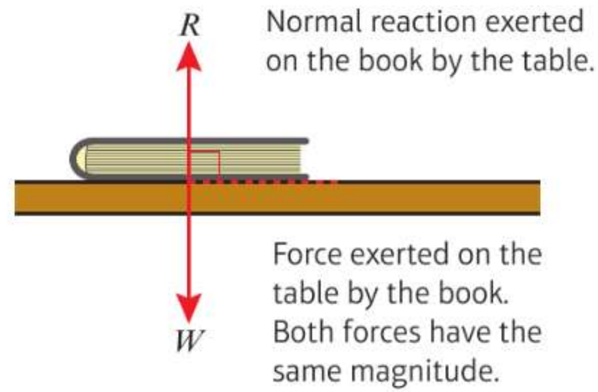
Do this one step at a time.

?

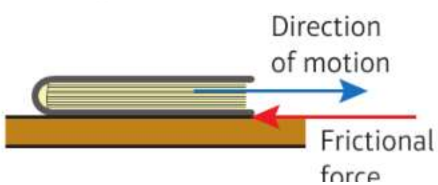
Types of Force and Force Diagrams

You will encounter a variety of forces in mechanics. These **force diagrams** show some of the most common forces.

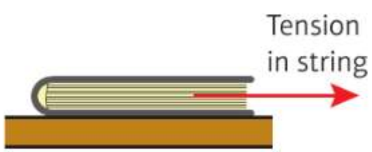
- The **weight** (or gravitational force) of an object acts vertically downwards.
- The **normal reaction** is the force which acts perpendicular to a surface when an object is in contact with the surface. In this example the normal reaction is due to the weight of the book resting on the surface of the table.



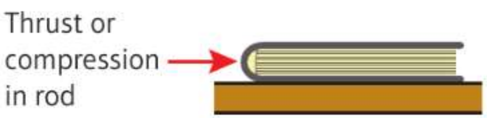
- The **friction** is a force which opposes the motion between two rough surfaces.



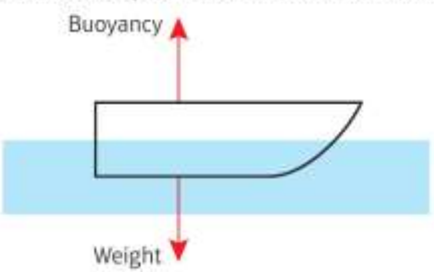
- If an object is being pulled along by a string, the force acting on the object is called the **tension** in the string.



- If an object is being pushed along using a light rod, the force acting on the object is called the **thrust** or **compression** in the rod.



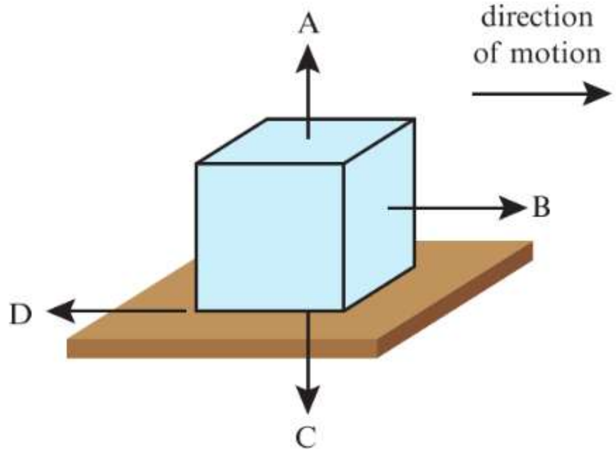
- **Buoyancy** is the upward force on a body that allows it to float or rise when submerged in a liquid. In this example buoyancy acts to keep the boat afloat in the water.



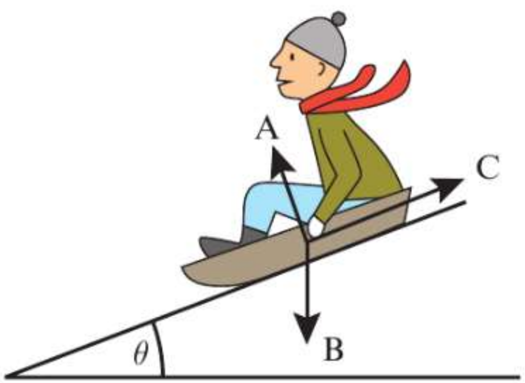
- **Air resistance** opposes motion. In this example the weight of the parachutist acts vertically downwards and the air resistance acts vertically upwards.

Write down the names of the forces shown in each of these diagrams.

A box being pushed along rough ground



A man sliding down a hill on a sledge



Exercise 8C

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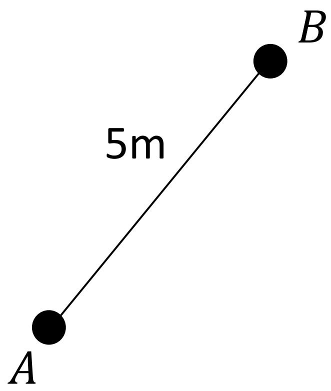
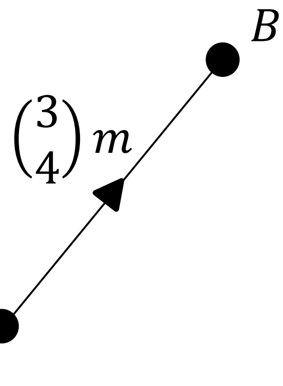
Vectors ↔ Scalars

In Mechanics you will often need to convert to/from the scalar form of a quantity and the vector form.





A scalar quantity has magnitude (i.e. size) only.
The 5m is a distance.
The value is always positive.

A vector quantity also has direction.
The vector equivalent of distance is displacement.

Scalar Form	Vector Form
Distance	?
Speed	?

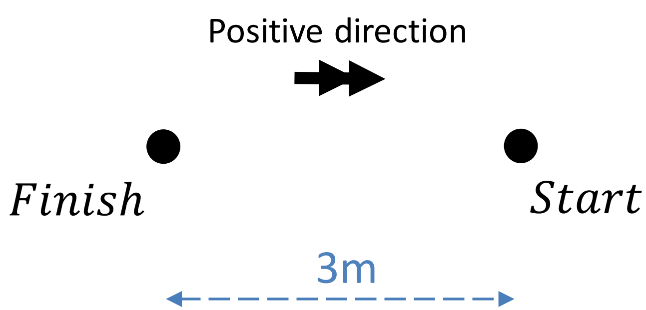
Other quantities which can be vectors or scalars:

?

Quantities which can only be scalars:

?

Note: 1-dimensional vectors are still different from scalars. Consider the displacement on a 1-dimensional line in a particular direction. If we'd gone backwards 3 units…

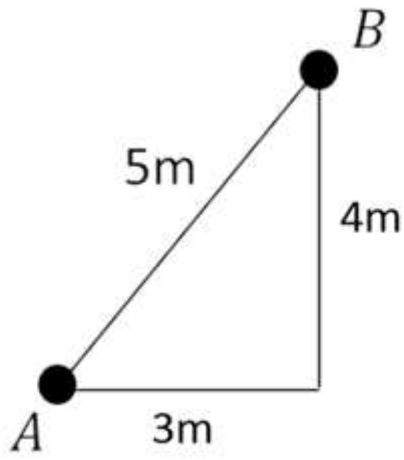


Distance: ?
Displacement: ?

Note: we don't write the brackets around 1D vectors. So 1D vectors look like scalars, except they're allowed to be positive or negative.

Vector Notation

In Mechanics you will often need to convert to/from the scalar form of a quantity and the vector form.



Column Notation

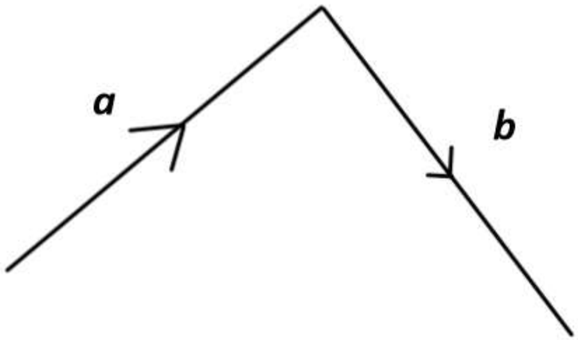
?

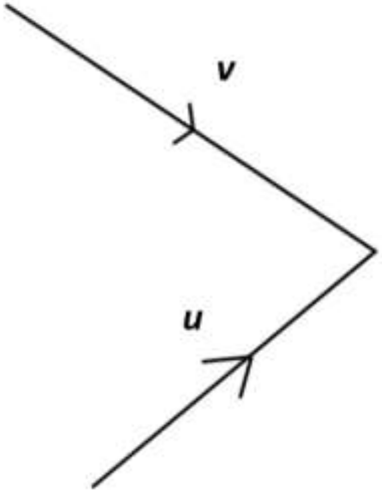
i-j Notation

(i and j are unit vectors of length 1)

?

Vector Addition and Subtraction

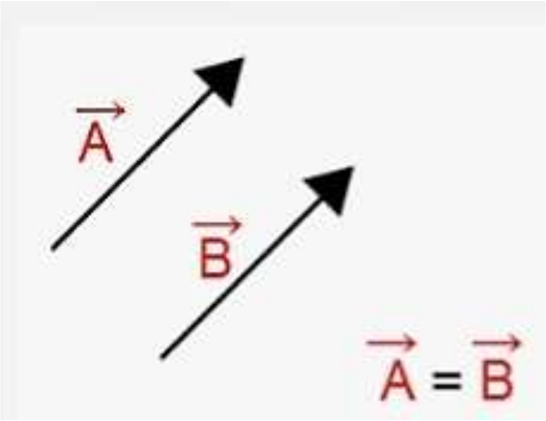




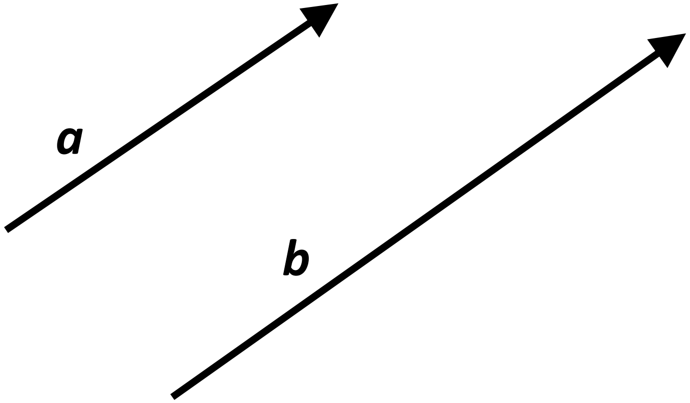
a + ***b***

u - ***v***

Two vectors are equal if they have the same direction and magnitude.



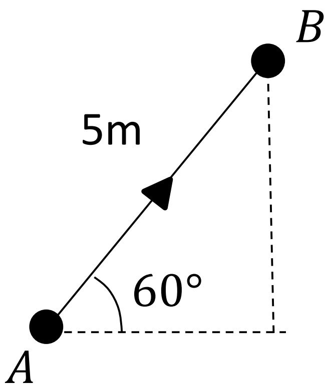
Two vectors are parallel if they have the **same direction but different magnitudes.**



$$\boldsymbol{a} = \lambda \boldsymbol{b}$$

Vectors ↔ Scalars

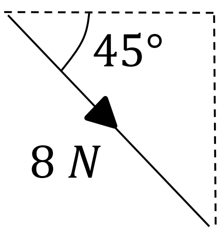
Scalar Form	Vector Form



?

To convert to vector form, just use basic trigonometry to find the x-change and y-change.

Speed Tip: If x is the magnitude, use $x \cos \theta$ for the side adjacent to the angle and $x \sin \theta$ for the side opposite it.



?

In the y-direction the force is acting downwards.

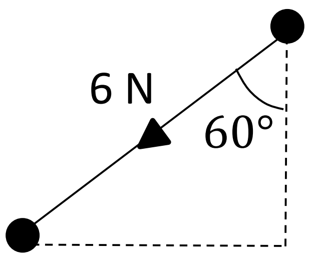
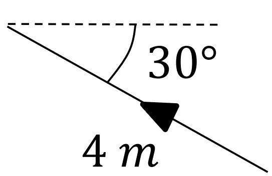
?

Velocity:

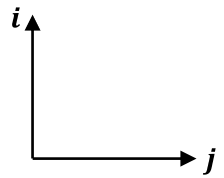
$$\begin{pmatrix} 5 \\ -12 \end{pmatrix} ms^{-1}$$

To convert scalar form, just find the **magnitude** of the vector using Pythagoras.

Further Examples

Scalar Form	Vector Form
	?
?	$(6\boldsymbol{i} - 8\boldsymbol{j})\ ms^{-2}$
	?

Recall from Pure Year 1 that $6\boldsymbol{i} - 8\boldsymbol{j}$ is another way of writing $\binom{6}{-8}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the positive x and y directions.



Test Your Understanding

A woman walks from A to B and then from B to C.
Her displacement from A to B is $6\underline{i} + 4\underline{j}$ m.
Her displacement from B to C is $5\underline{i} - 12\underline{j}$ m.

a) What is the magnitude of the displacement from A to C?
b) What is the total distance the woman has walked in getting from A to C?

? Suitable Diagram

a

?

b

?

Exercise 8D

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