MechYr2 Chapter 8 :: Further Kinematics

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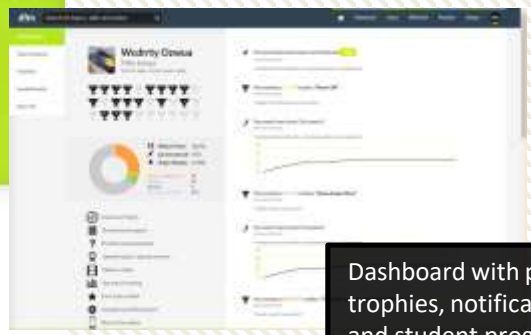

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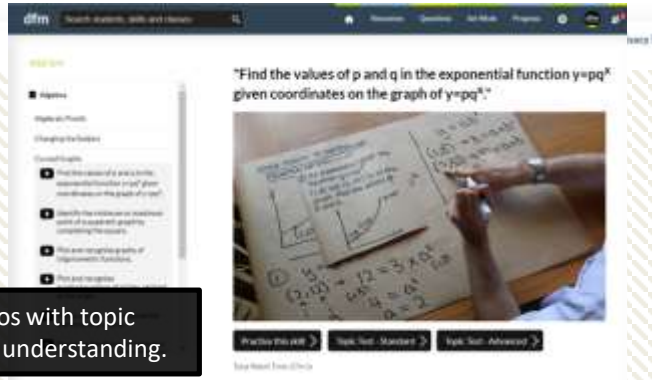

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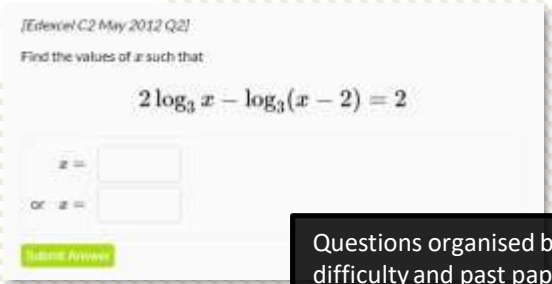


Questions organised by topic, difficulty and past paper.

Overview

This chapter concerns how can use **vectors to represent motion**. In the case of constant acceleration, can we still use our 'suvat' equations? And what if we have variable acceleration with expressions in terms of t?

1:: Vector equations for motion.

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$ **(3)**

(b) Find the bearing on which P is moving when $t = 2$ **(2)**

(c) Find the value of t when P is moving

(i) parallel to $\mathbf{j}$,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$. **(6)**

2:: Variable acceleration with vectors.

"A particle P of mass 0.8kg is acted on by a single force $\boldsymbol{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where

$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$$

Find (a) the speed of P when $t = 4$

(b) The acceleration of P as a vector when $t = 2$

(c) $\boldsymbol{F}$ when $t = 2$."

3:: Integration with vectors to find velocity/displacement

"A particle P is moving in a plane. At time t seconds, its velocity $\boldsymbol{v}$ ms^{-1} is given by $\boldsymbol{v} = 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}, \quad t \geq 0$

When $t = 0$, the position vector of P with respect to a fixed origin O is $(2\boldsymbol{i} - 3\boldsymbol{j})$ m. Find the position vector of P at time t seconds."

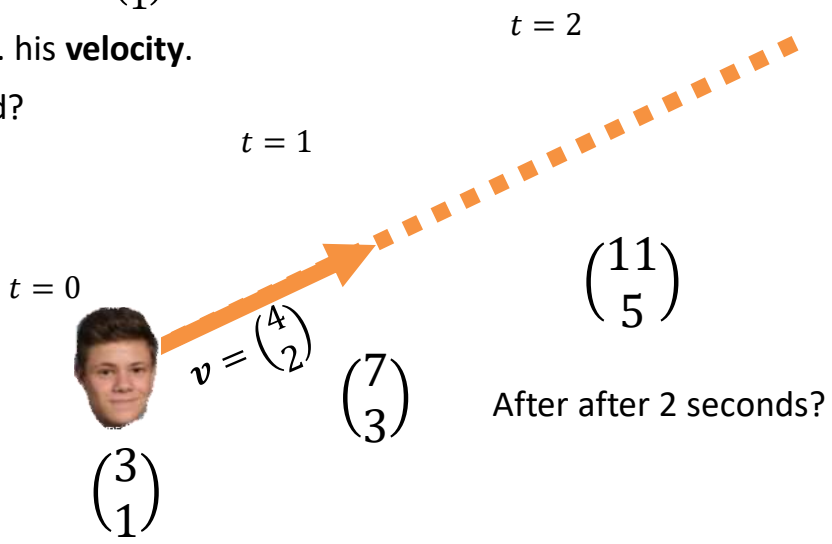
Note for teachers: The first item was in the old M1 with projectile motion in M2. Variable acceleration was in M2.

Vector motion

Initially, Lewis is at the position vector $\begin{pmatrix}3\\1\end{pmatrix}$.

Each second, he moves $\begin{pmatrix}4\\2\end{pmatrix}$, i.e. his **velocity**.

Where will he be after 1 second?



After after 2 seconds?

So in general, where would Lewis be after t seconds, in terms of t?

?

Position vector $\boldsymbol{r}$ of particle:

$$\boldsymbol{r} = \boldsymbol{r_0} + \boldsymbol{v}t$$

where r_0 is initial position and $\boldsymbol{v}$ is velocity.

Fro Note: I don't really remember as a formula as such though, but as 'common sense' using the reasoning above.

Fro Note II: Further Mathematicians who have finished Vectors in Core Pure Yr1 may see the similarities with vector equations of straight lines.

Example

At time $t = 0$, where t is the time (in seconds), a particle is at the point with position vector $(4\boldsymbol{i} - \boldsymbol{j})$ m and travels with velocity $(-2\boldsymbol{i} + 2\boldsymbol{j})$ ms^{-1}. Find:
a) The position vector of the particle after t seconds
b) The distance the particle is from the origin, O, after 3 seconds.

a

?

b

?

? Diagram

Fropinion: I prefer to avoid the $\boldsymbol{i}$ and $\boldsymbol{j}$ notation and write in conventional vector form. They're easier to manipulate.

Example

A particle starts at a point 8m from O at an angle of 45^{O} anti-clockwise from east and travels with a velocity $(-2\boldsymbol{i} - 3\boldsymbol{j})$ ms^{-1}, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors due east and north respectively.
Find the position vector of the particle after t seconds in the form $\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v}$.

?

? Diagram

$suvat$… but with vectors!

Some $suvat$ equations work with vectors. By convention, we use $\boldsymbol{r}$ instead of $\boldsymbol{s}$ for displacement in 2D/3D (as we did in the previous exercise). In 2D, which of the quantities are vectors and which are scalars?

$$\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$r =$?
$u =$?
$v =$?
$a =$?
$t =$?

Note that as $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors, we can't for example use $v^2 = u^2 + 2as$, as you can't square a vector.

A particle is initially travelling with velocity $(-2\boldsymbol{i} - 9\boldsymbol{j})$ ms^{-1} and 2 seconds later it has a velocity of $(6\boldsymbol{i} - 11\boldsymbol{j})$ ms^{-1}, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the directions of the positive x- and y- axes respectively. Given that the acceleration of the particle is constant, find:

a) The acceleration

b) The magnitude of the acceleration

c) The angle that the acceleration makes with the vector $\boldsymbol{j}$

a ?

b ?

c ?

Example

[Textbook] An ice skater is skating on a large flat ice rink. At time $t = 0$ the skater is at a fixed point O and is travelling with velocity $(2.4\boldsymbol{i} - 0.6\boldsymbol{j})$ ms^{-1}.
At time $t = 20$ s the skater is travelling with velocity $(-5.6\boldsymbol{i} + 3.4\boldsymbol{j})$ ms^{-1}.
Relative to O, the skater has position vector $\boldsymbol{s}$ at time t seconds.
Modelling the ice skater as a particle with constant acceleration, find:
(a) The acceleration of the ice skater
(b) An expression for $\boldsymbol{s}$ in terms of t
(c) The time at which the skater is directly north-east of O.
A second skater travels so that she has position vector $\boldsymbol{r} = (1.1t - 6)\boldsymbol{j}$ m relative to O at time t.
(d) Show that the two skaters will meet.

a

?

b

?

c

?

d

?

Test Your Understanding

Edexcel M1(Old) May 2013(R) Q6

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O.*]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j})$ km.

(a) Find the position vector of S at time t hours. **(2)**

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j})$ km h^{-1}. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j})$ km. The two ships meet at the point P.

(b) Find the value of n. **(5)**

(c) Find the distance OP. **(4)**

(a) ?

(b) ?

(c) ?

Exercise 8A

Pearson Stats/Mechanics Year 2
Pages 162-164

Vector methods for projectiles

Previously we considered the initial speed of the projectile and the angle of projection. But we could also **use a velocity vector to represent the initial projection** (vectors have both direction and magnitude) and subsequent motion.

A ball is projected from the origin with velocity $(12\boldsymbol{i} + 24\boldsymbol{j})$ms^{-1} where $\boldsymbol{i}$ and $\boldsymbol{j}$ are horizontal and vertical unit vectors respectively. The particle moves freely under gravity. Find:

a) The position vector of the ball after 3s

b) The speed of the ball after 3s

c) The ball strikes the ground at point B. Determine the distance OB

b

?

? Diagram

c

?

a

?

Vector methods for projectiles

A particle *P* is projected with velocity $(4p\boldsymbol{i} + 5p\boldsymbol{j})$ ms^{-1} from a point *O* on a horizontal plane, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are horizontal and vertical unit vectors respectively.
The particle *P* strikes the plane at the point *A,* which is 800 m from *O*.

a) Show that $p = 14$.

b) Find the time of flight from *O* to *A*.

The particle *P* passes through a point *B* with speed 60 m s^{-1}.

c) Find the height of *B* above the horizontal plane.

a

?

? Diagram

Vector methods for projectiles

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The particle *P* passes through a point *B* with speed 60 m s^{-1}.

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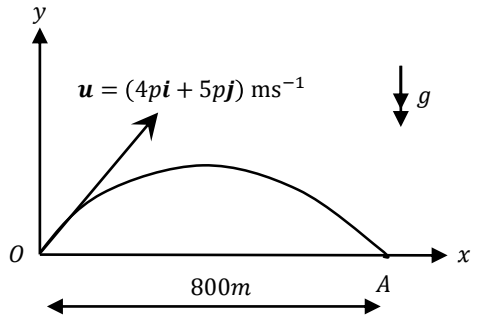


b

?

c

?

Test Your Understanding

Edexcel M2(Old) Jan 2012 Q7

[*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively.*]

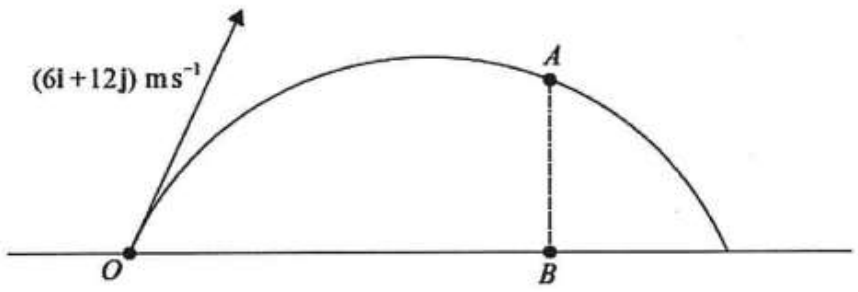


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

(3)

(a) ?

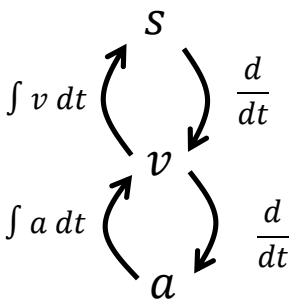
(b) ?

(c) ?

Exercise 8B

Pearson Stats/Mechanics Year 2
Pages 166-167

Variable Acceleration in One Dimension



In Mechanics Yr1 we saw that velocity was the rate of change of displacement, and thus $v = \frac{ds}{dt}$. Similarly acceleration is the rate of change of velocity, and thus $a = \frac{dv}{dt}$

Let's stick to one-dimension for the moment, but you may need to **differentiate more complex functions of *t* that use Pure Year 2 techniques**.

[Textbook] A particle is moving in a straight line with acceleration at time t seconds given by

$$a = \cos 2\pi t \text{ ms}^{-2}, \qquad t \geq 0$$

The velocity of the particle at time $t = 0$ is $\frac{1}{2\pi}$ ms^{-1}. Find:

(a) an expression for the velocity at time t seconds
(b) the maximum speed
(c) the distance travelled in the first 3 seconds.

a ?

b ?

c ?

Test Your Understanding

[Textbook] A particle of mass 6kg is moving on the positive x-axis. At time t seconds the displacement, s, of the particle from the origin is given by

$$s = 2t^{\frac{3}{2}} + \frac{e^{-2t}}{3} \text{ m}, \qquad t \geq 0$$

(a) Find the velocity of the particle when $t = 1.5$.

Given that the particle is acted on by a single force of variable magnitude F N which acts in the direction of the positive x-axis,

(b) Find the value of F when $t = 2$

a

?

b

?

Recap: Due to the chain rule,

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

Exercise 8C

Pearson Stats/Mechanics Year 2
Pages 168-170

Differentiating Vectors

Suppose that $\boldsymbol{v} = \begin{pmatrix} t^2 \\ \sin t \end{pmatrix}$. What would be the acceleration?

?

If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ then $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \dot{\boldsymbol{r}} = \dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j}$

and $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d^2\boldsymbol{r}}{dt} = \ddot{\boldsymbol{r}} = \ddot{x}\boldsymbol{i} + \ddot{y}\boldsymbol{j}$

Notational note: Dot notation is a short-hand for differentiation with respect to time: $\dot{x} = \frac{dx}{dt}$
Its use is common in Physics.

[Textbook] A particle P of mass 0.8kg is acted on by a single force $\mathbf{F}$ N. Relative to a fixed origin O, the position vector of P at time t seconds is $\boldsymbol{r}$ metres, where

$$\boldsymbol{r} = 2t^3\boldsymbol{i} + 50t^{-\frac{1}{2}}\boldsymbol{j}, \qquad t \geq 0$$

Find:

(a) the speed of P when $t = 4$

(b) the acceleration of P as a vector when $t = 2$

(c) $\mathbf{F}$ when $t = 2$.

a ?

b ?

c ?

Exercise 8D

Pearson Stats/Mechanics Year 2
Pages 171-173

Integrating Vectors

We can similarly integrate the $\boldsymbol{i}$ and $\boldsymbol{j}$ components to get from acceleration to velocity and velocity to displacement.

A force $\boldsymbol{F}$ acts on a body of mass 250g which is initially at rest at a fixed point O. If $\boldsymbol{F} = ((5t - 2)\boldsymbol{i} + 4t\boldsymbol{j})$N, where t is the time for which the force has been acting on the body, find expressions for:

a) The velocity vector of the body at time t.
b) The position vector of the body at time t.

a

?

b

?

Further Example

[Textbook] A particle P is moving in a plane so that, at time t seconds, its acceleration is $(4\boldsymbol{i} - 2t\boldsymbol{j})$ ms^{-2}. When $t = 3$, the velocity of P is $6\boldsymbol{i}$ ms^{-1} and the position vector of P is $(20\boldsymbol{i} + 3\boldsymbol{j})$ m with respect to a fixed origin O. Find:

(a) the angle between the direction of motion of P and $\boldsymbol{i}$ when $t = 2$

(b) the distance of P from O when $t = 0$.

a

?

b

?

Test Your Understanding

Edexcel M2(Old) Jan 2013 Q4

At time t seconds the velocity of a particle P is $[(4t - 5)\mathbf{i} + 3\mathbf{j}]$ m s^{-1}. When $t = 0$, the position vector of P is $(2\mathbf{i} + 5\mathbf{j})$ m, relative to a fixed origin O.

(a) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(b) Find an expression for the position vector of P at time t seconds.

(4)

A second particle Q moves with constant velocity $(-2\mathbf{i} + c\mathbf{j})$ m s^{-1}. When $t = 0$, the position vector of Q is $(11\mathbf{i} + 2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i} + 14\mathbf{j})$ m.

(c) Find

(i) the value of c,

(ii) the value of d.

(5)

(a) ?

(b) ?

(c) ?

Exercise 8E

Pearson Stats/Mechanics Year 2
Pages 175-176

You have reached the end of maths.*

* At A Level.