MechYr1 Chapter 11 :: Variable Acceleration

jfrost@tiffin.kingston.sch.uk

www.drfrostmaths.com
@DrFrostMaths

Last modified: 5th August 2018

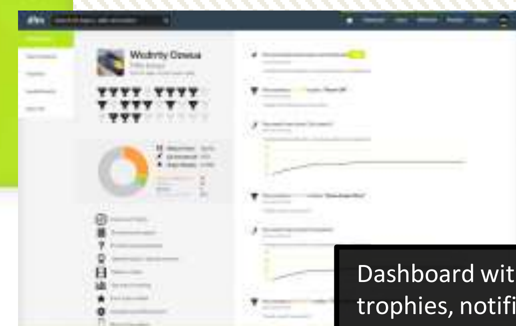
www.drfrostmaths.com

Everything is **completely free**.
Why not register?

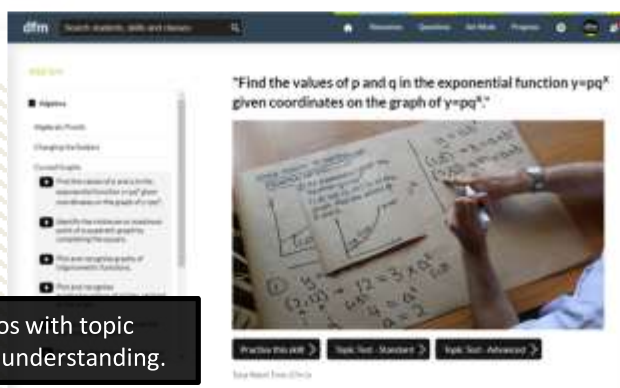
Register now to interactively practise questions on this topic, including past paper questions and extension questions (including MAT + UKMT). Teachers: you can create student accounts (or students can register themselves), to set work, monitor progress and even create worksheets.

With questions by:

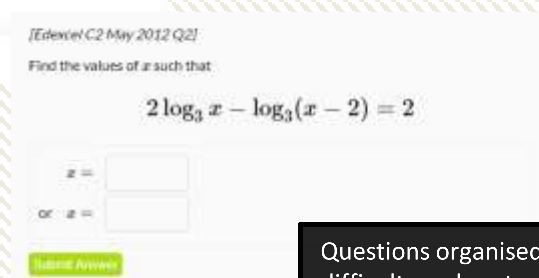
edexcel
OCR
AQA
UKMT



Dashboard with points, trophies, notifications and student progress.



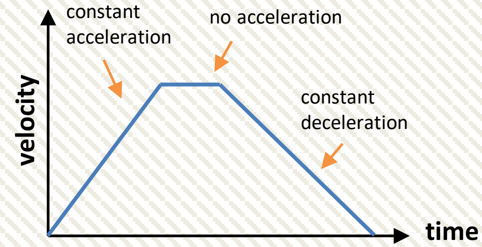
Teaching videos with topic tests to check understanding.



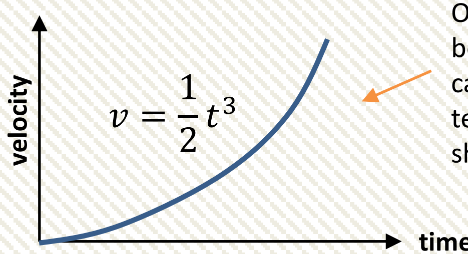
Questions organised by topic, difficulty and past paper.

Functions of time

Up to now, the acceleration has always been constant in any particular period of time…



However, it's possible to specify either the displacement, velocity or acceleration as any function of time (i.e. an expression in terms of t). This allows the acceleration to constantly change.



Our velocity-time graph can be any shape we want! We can use an expression in terms of t to give a certain shape.

The velocity-time graph of a body is shown above, where $v = \frac{1}{2}t^3$.

(a) What is the velocity after 4 seconds have elapsed?

(b) How many seconds have elapsed when the velocity of the body is 108 ms^{-1}?

a ?

b ?

[Textbook] A body moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v = 2t^2 - 16t + 24$. Find

(a) The initial velocity

(b) The values of t when the body is instantaneously at rest.

(c) The value of t when the velocity is 64 ms^{-1}.

(d) The greatest speed of the body in the interval $0 \leq t \leq 5$.

a ?

b ?

c ?

d ?

Exercise 11A

Pearson Stats/Mechanics Year 1
Page 184

Classes in a rush (or Further Mathematicians) may wish to skip this exercise.

Using Differentiation

In Chapter 9, we saw that velocity v is the rate of change of displacement s (i.e. the gradient). But in Pure, we know that we can use differentiation to find the gradient function:

$v =$?

velocity is the rate of change of displacement

and similarly…

$a =$?

acceleration is the rate of change of velocity

Memory Tip: I picture interchanging between s, v, a as differentiating to go downwards and integrating to go upwards:

s
$\int v\, dt$ $\quad$ $\frac{d}{dt}$
v
$\int a\, dt$ $\quad$ $\frac{d}{dt}$
a

(We will do integration a bit later)

A body moves in a straight line such that $v = 2t^2 - 11t + 14$. Initially (i.e. when $t = 0$), the displacement of the body from some fixed point O on the line is 50m. Find:

a) The initial velocity of the body
b) The values of t when the body is at rest
c) The acceleration of the body when t = 5s
d) The displacement of the body when t = 6s (we cover integration later)

a ?

b ?

c ?

d ?

Test Your Understanding

Pudding the Cat's displacement from a house, in metres, is $t^3 - \frac{3}{2}t^2 - 36t$ where t is in seconds.
(a) Determine the velocity of the cat when $t = 2$.
(b) At what time will the cat be instantaneously at rest?
(c) What is the cat's acceleration after 5 seconds?

a

?

b

?

c

?

Key Phrases

- At rest: $v = 0$
- Returns to starting position: $s = 0$
- Constant Velocity: $a = 0$
- Total Distance: Area beneath graph - Integrate

Exercise 11B

Pearson Stats/Mechanics Year 1

Page 185-186

Maxima and Minima Problems

Recall from Pure that at minimum/maximum points, the gradient is 0. We could therefore for example find where the velocity is minimum/maximum by finding when $\frac{dv}{dt} = 0$ (i.e. when the acceleration is 0). Similarly, we can find the maximum and minimum values for displacement and acceleration.

A particle P, moves in a straight line such that its velocity, $v\ ms^{-1}$ at time $t\ s$, is given by:

$$v = 5 - 9t + 6t^2 - t^3 \qquad \text{where } 0 \leq t \leq 4$$

a) Find the difference between the maximum and minimum velocities over this time interval

b) Sketch a velocity-time graph for the motion of P

c) Find the maximum acceleration over this time interval

a

?

b

?

c

?

Test Your Understanding

A dolphin escapes from Seaworld and its velocity as it speeds away from the park, is $t^3 - 9t^2 - 48t + 500$ (in ms^{-1}), until it reaches its maximum velocity, and then subsequently remains at this velocity.

(a) When does the dolphin reach its maximum velocity?

(b) What is this maximum velocity?

a

?

b

?

Edexcel M2 June 2013 Q3a,b

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

Find

(*a*) the times when P is instantaneously at rest,

(*b*) the greatest speed of P in the interval $0 \leq t \leq 4$,

a

?

b

?

Test Your Understanding

A particle P, moves in a straight line. After t seconds, its distance, s m from its starting point A, when t = 0, is given by:

$$s = 2t^3 - 9t^2 + 12t \quad \text{where } t \geq 0$$

a) Show that the particle never returns to its starting point
b) Find the distances from A at which the particle is instantaneously at rest
c) Find the acceleration of the particle at time t = 3s

a

?

c

?

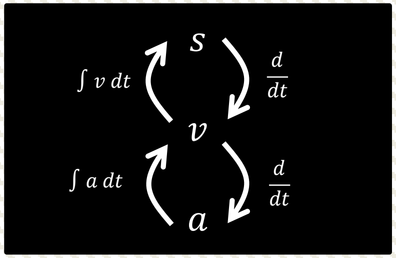
b

?

Exercise 11C

Pearson Stats/Mechanics Year 1
Page 187-188

Using Integration



Differentiating (with respect to time) gets us from displacement to velocity, and from velocity to acceleration.
So naturally, integrating (with respect to time) gets us from acceleration to velocity, and from velocity to displacement. As mentioned earlier, it's helpful to picture the graph on the left, where we move down to differentiate and up to integrate.

A particle P, moves in a straight line. At t seconds its acceleration is $(6t + 12)ms^{-1}$. When $t = 0$, P is at the point A and its velocity is $3ms^{-1}$.
a) Find an expression for the velocity of P in terms of t
b) Find the distance travelled between times $t = 3$ and $t = 5$

a

?

b

?

Further Example

[Textbook] A particle travels in a straight line. After t seconds its velocity, v ms^{-1}, is given by $v = 5 - 3t^2$, $t \geq 0$. Find the distance travelled by the particle in the third second of its motion.

?

Test Your Understanding

Edexcel M2 June 2015 Q6

A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)$ m s^{-1}. When $t = 0$, P is 15 m from the origin O.

Find

(*a*) the values of t when P is instantaneously at rest, **(3)**

(*b*) the acceleration of P when $t = 5$, **(3)**

(*c*) the total distance travelled by P in the interval $0 \leq t \leq 5$. **(5)**

Warning: recall that if the curve goes above and below the x-axis, we need to find each area separately.

a

?

b

?

c

?

Exercise 11D

Pearson Stats/Mechanics Year 1
Page 189-191

Constant acceleration formulae

In Chapter 9, we work out the various $suvat$ formulae by using a velocity-time graph. But it's also possible to derive all of these using integration, provided that we consider that **acceleration is constant**.

Given a body has constant acceleration a, initial velocity u and its initial displacement is 0 m, prove that:

(a) Final velocity: $v = u + at$

(b) Displacement: $s = ut + \frac{1}{2}at^2$

a

?

Note that a is a **constant** not a variable, so integrates as such.
Just as $\int 3\, dx = 3x + c$, we find $\int a\, dt = at + c$

b

?

Again, because u is fixed, we can treat it as a constant.

Exercise 11E

Pearson Stats/Mechanics Year 1
Pages 192-193

The End