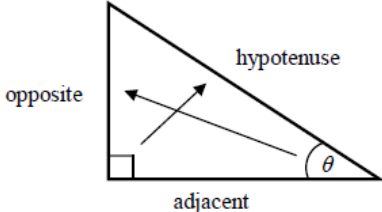


Trigonometry in right-angled triangles

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

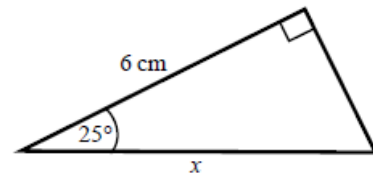
Key points

- In a right-angled triangle:
 - the side opposite the right angle is called the hypotenuse
 - the side opposite the angle θ is called the opposite
 - the side next to the angle θ is called the adjacent.
- 
- In a right-angled triangle:
 - the ratio of the opposite side to the hypotenuse is the sine of angle θ , $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
 - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ , $\cos \theta = \frac{\text{adj}}{\text{hyp}}$
 - the ratio of the opposite side to the adjacent side is the tangent of angle θ , $\tan \theta = \frac{\text{opp}}{\text{adj}}$
 - If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: \sin^{-1} , \cos^{-1} , \tan^{-1} .
 - The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

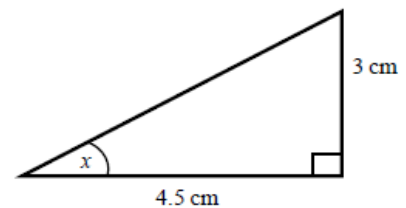
Examples

Example 1 Calculate the length of side x .
Give your answer correct to 3 significant figures.



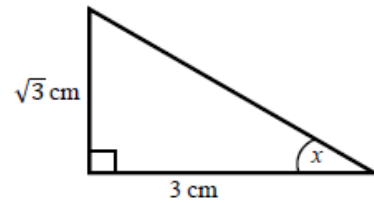
<p> $\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\cos 25^\circ = \frac{6}{x}$ $x = \frac{6}{\cos 25^\circ}$ $x = 6.620\ 267\ 5\dots$ $x = 6.62\ \text{cm}$ </p>	<ol style="list-style-type: none"> 1 Always start by labelling the sides. 2 You are given the adjacent and the hypotenuse so use the cosine ratio. 3 Substitute the sides and angle into the cosine ratio. 4 Rearrange to make x the subject. 5 Use your calculator to work out $6 \div \cos 25^\circ$. 6 Round your answer to 3 significant figures and write the units in your answer.
--	--

Example 2 Calculate the size of angle x .
Give your answer correct to 3 significant figures.



<p> $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{3}{4.5}$ $x = \tan^{-1}\left(\frac{3}{4.5}\right)$ $x = 33.690\ 067\ 5\dots$ $x = 33.7^\circ$ </p>	<ol style="list-style-type: none"> 1 Always start by labelling the sides. 2 You are given the opposite and the adjacent so use the tangent ratio. 3 Substitute the sides and angle into the tangent ratio. 4 Use \tan^{-1} to find the angle. 5 Use your calculator to work out $\tan^{-1}(3 \div 4.5)$. 6 Round your answer to 3 significant figures and write the units in your answer.
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Example 3 Calculate the exact size of angle x .

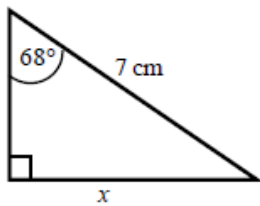


<p> $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30^\circ$ </p>	<ol style="list-style-type: none"> 1 Always start by labelling the sides. 2 You are given the opposite and the adjacent so use the tangent ratio. 3 Substitute the sides and angle into the tangent ratio. 4 Use the table from the key points to find the angle.
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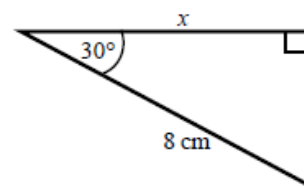
Practice

1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

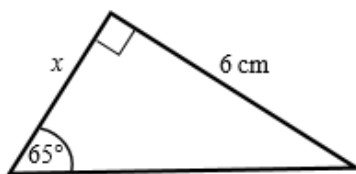
a



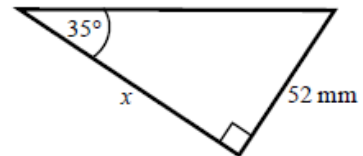
b



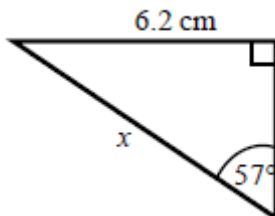
c



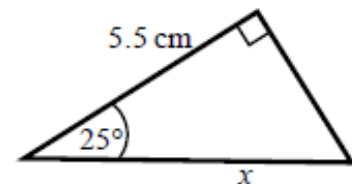
d



e

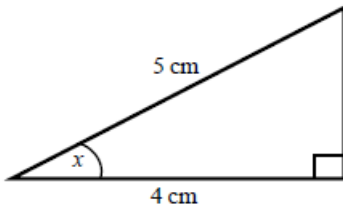


f

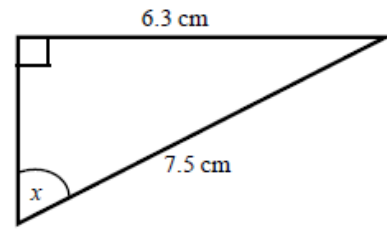


- 2 Calculate the size of angle x in each triangle. Give your answers correct to 1 decimal place.

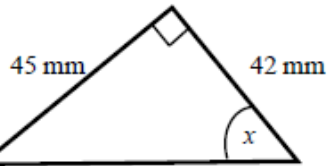
a



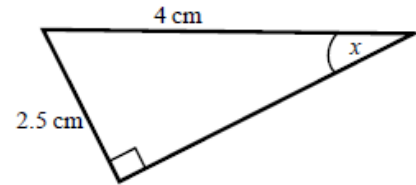
b



c



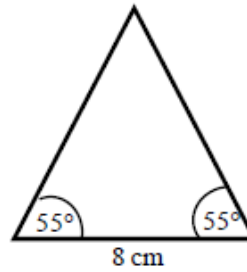
d



- 3 Work out the height of the isosceles triangle. Give your answer correct to 3 significant figures.

Hint:

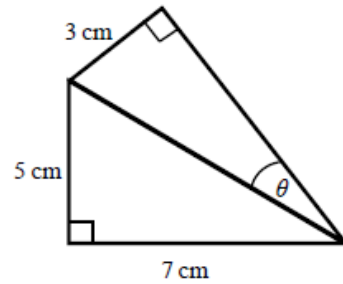
Split the triangle into two right-angled triangles.



- 4 Calculate the size of angle θ . Give your answer correct to 1 decimal place.

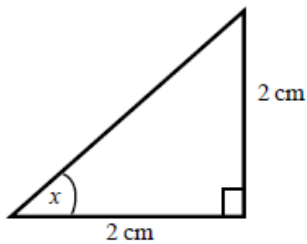
Hint:

First work out the length of the common side to both triangles, leaving your answer in surd form.

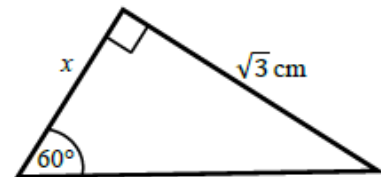


- 5 Find the exact value of x in each triangle.

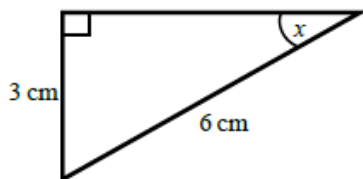
a



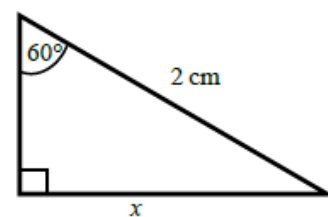
b



c



d



The cosine rule

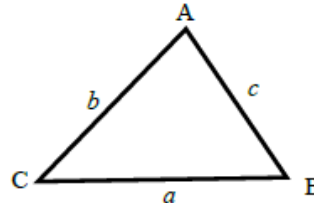
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.1 The cosine rule

Key points

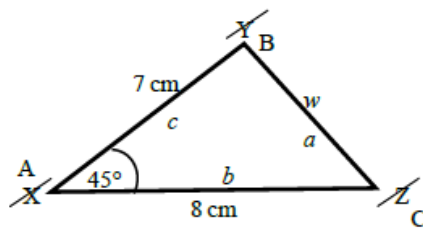
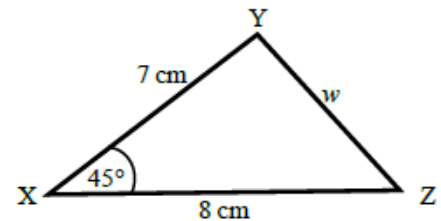
- a is the side opposite angle A .
- b is the side opposite angle B .
- c is the side opposite angle C .



- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc \cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

Examples

Example 4 Work out the length of side w .
Give your answer correct to 3 significant figures.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45^\circ$$

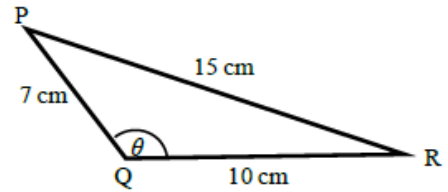
$$w^2 = 33.804\ 040\ 51\dots$$

$$w = \sqrt{33.80404051}$$

$$w = 5.81 \text{ cm}$$

- 1 Always start by labelling the angles and sides.
- 2 Write the cosine rule to find the side.
- 3 Substitute the values a , b and A into the formula.
- 4 Use a calculator to find w^2 and then w .
- 5 Round your final answer to 3 significant figures and write the units in your answer.

Example 5 Work out the size of angle θ .
Give your answer correct to 1 decimal place.

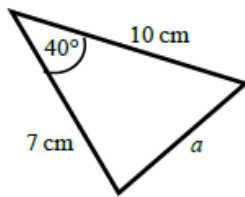


$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$ $\cos \theta = \frac{-76}{140}$ $\theta = 122.878\ 349\dots$ $\theta = 122.9^\circ$	<ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the cosine rule to find the angle. 3 Substitute the values a, b and c into the formula. 4 Use \cos^{-1} to find the angle. 5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$. 6 Round your answer to 1 decimal place and write the units in your answer.
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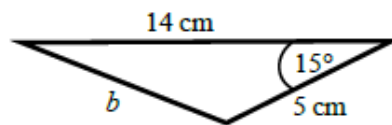
Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

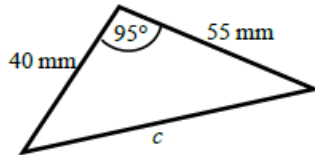
a



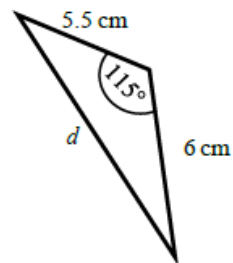
b



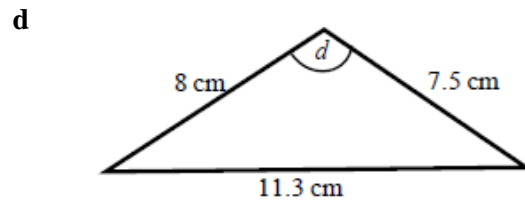
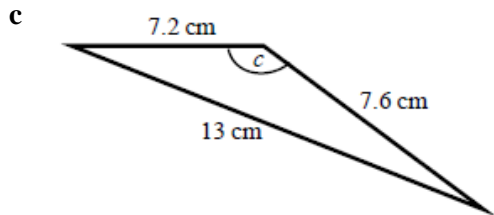
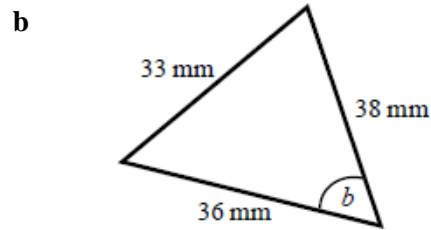
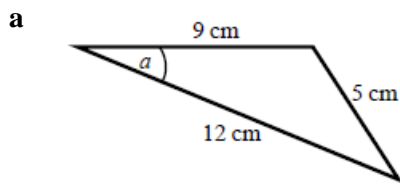
c



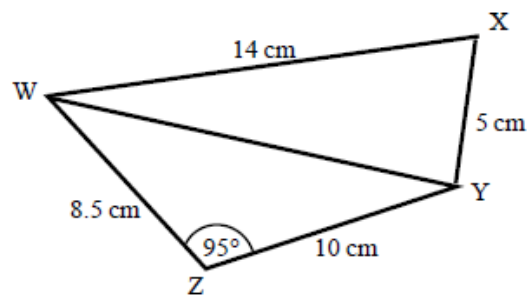
d



- 7 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.



- 8
- a Work out the length of WY. Give your answer correct to 3 significant figures.
- b Work out the size of angle WXY. Give your answer correct to 1 decimal place.



The sine rule

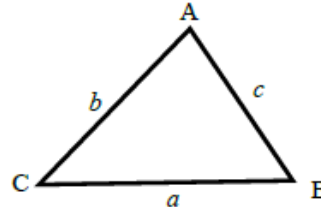
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.2 The sine rule

Key points

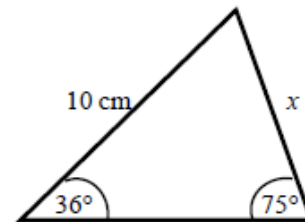
- a is the side opposite angle A .
- b is the side opposite angle B .
- c is the side opposite angle C .



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

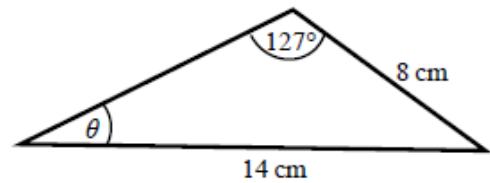
Examples

Example 6 Work out the length of side x .
Give your answer correct to 3 significant figures.



$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 36^\circ} = \frac{10}{\sin 75^\circ}$ $x = \frac{10 \times \sin 36^\circ}{\sin 75^\circ}$ $x = 6.09 \text{ cm}$	<ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the sine rule to find the side. 3 Substitute the values a, b, A and B into the formula. 4 Rearrange to make x the subject. 5 Round your answer to 3 significant figures and write the units in your answer.
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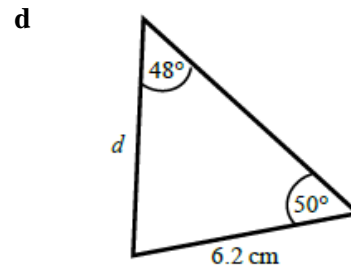
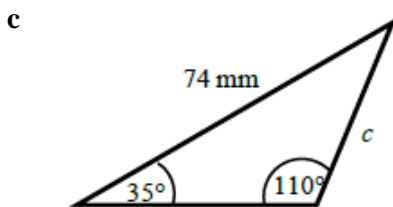
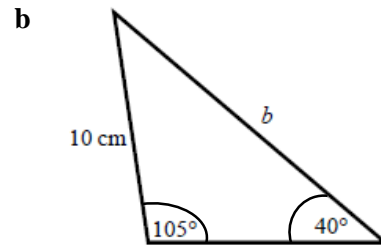
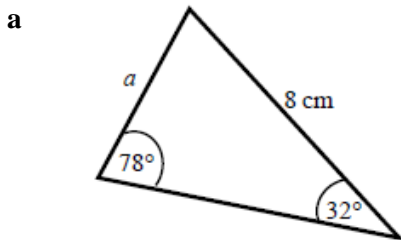
Example 7 Work out the size of angle θ .
Give your answer correct to 1 decimal place.



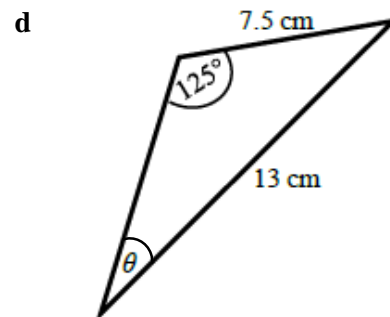
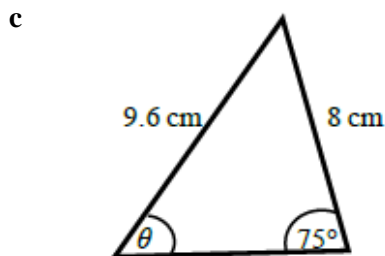
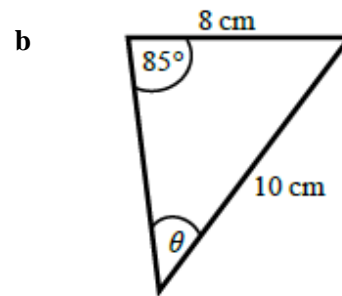
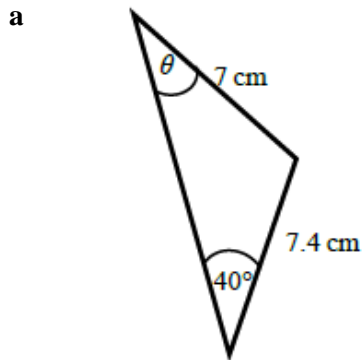
$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{8} = \frac{\sin 127^\circ}{14}$ $\sin \theta = \frac{8 \times \sin 127^\circ}{14}$ $\theta = 27.2^\circ$	<ol style="list-style-type: none"> 1 Always start by labelling the angles and sides. 2 Write the sine rule to find the angle. 3 Substitute the values a, b, A and B into the formula. 4 Rearrange to make $\sin \theta$ the subject. 5 Use \sin^{-1} to find the angle. Round your answer to 1 decimal place and write the units in your answer.
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Practice

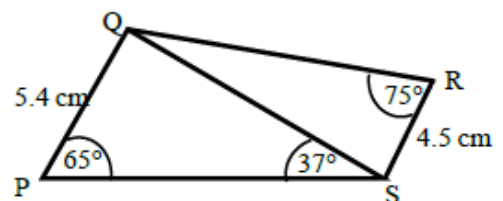
9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.



- 10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.



- 11 a Work out the length of QS.
Give your answer correct to 3 significant figures.
- b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.



Areas of triangles

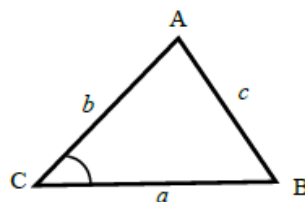
A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Textbook: Pure Year 1, 9.3 Areas of triangles

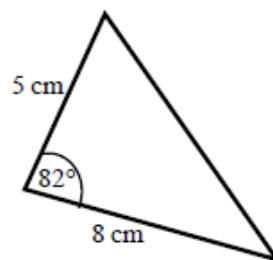
Key points

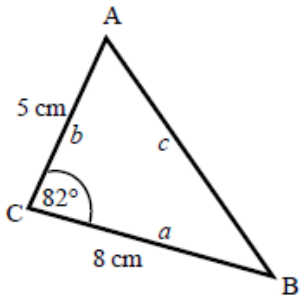
- a is the side opposite angle A .
 b is the side opposite angle B .
 c is the side opposite angle C .
- The area of the triangle is $\frac{1}{2}ab \sin C$.



Examples

Example 8 Find the area of the triangle.

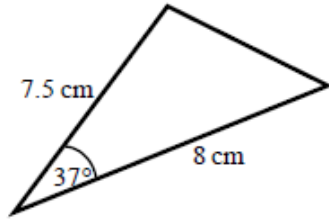


 <p>Area = $\frac{1}{2}ab \sin C$</p> <p>Area = $\frac{1}{2} \times 8 \times 5 \times \sin 82^\circ$</p> <p>Area = 19.805 361...</p> <p>Area = 19.8 cm²</p>	<ol style="list-style-type: none"> 1 Always start by labelling the sides and angles of the triangle. 2 State the formula for the area of a triangle. 3 Substitute the values of a, b and C into the formula for the area of a triangle. 4 Use a calculator to find the area. 5 Round your answer to 3 significant figures and write the units in your answer.
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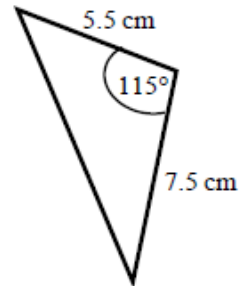
Practice

- 12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

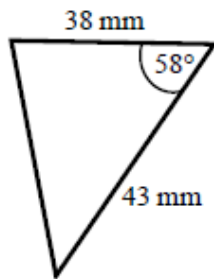
a



b



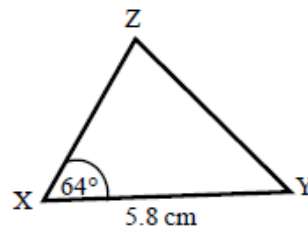
c



- 13 The area of triangle XYZ is 13.3 cm^2 .
Work out the length of XZ.

Hint:

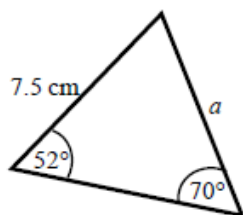
Rearrange the formula to make a side the subject.



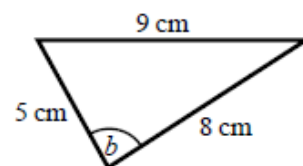
Extend

- 14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

a



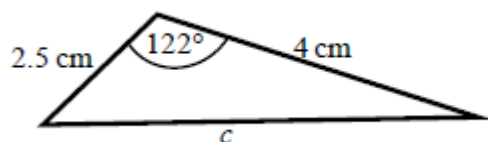
b



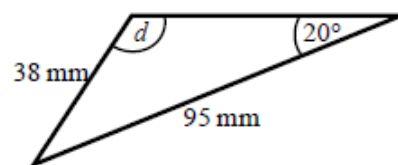
Hint:

For each one, decide whether to use the cosine or sine rule.

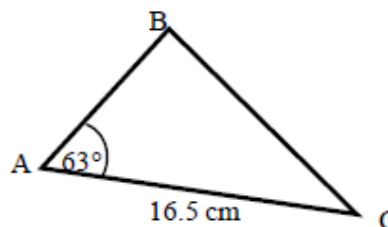
c



d



- 15 The area of triangle ABC is 86.7 cm^2 .
 Work out the length of BC.
 Give your answer correct to 3 significant figures.



Answers

- | | | | | |
|-----------|-------------------------------|-------------------------------|------------------------------|------------------------|
| 1 | a 6.49 cm | b 6.93 cm | c 2.80 cm | |
| | d 74.3 mm | e 7.39 cm | f 6.07 cm | |
| 2 | a 36.9° | b 57.1° | c 47.0° | d 38.7° |
| 3 | 5.71 cm | | | |
| 4 | 20.4° | | | |
| 5 | a 45° | b 1 cm | c 30° | d $\sqrt{3}$ cm |
| 6 | a 6.46 cm | b 9.26 cm | c 70.8 mm | d 9.70 cm |
| 7 | a 22.2° | b 52.9° | c 122.9° | d 93.6° |
| 8 | a 13.7 cm | b 76.0° | | |
| 9 | a 4.33 cm | b 15.0 cm | c 45.2 mm | d 6.39 cm |
| 10 | a 42.8° | b 52.8° | c 53.6° | d 28.2° |
| 11 | a 8.13 cm | b 32.3° | | |
| 12 | a 18.1 cm ² | b 18.7 cm ² | c 693 mm ² | |
| 13 | 5.10 cm | | | |
| 14 | a 6.29 cm | b 84.3° | c 5.73 cm | d 58.8° |
| 15 | 15.3 cm | | | |