

Chapter 9 - Mechanics

Constant Acceleration

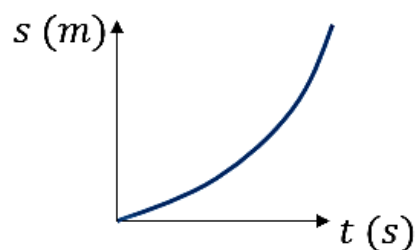
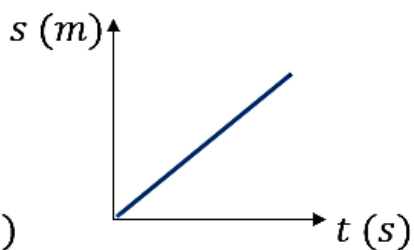
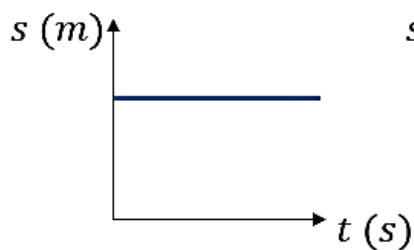
Chapter Overview

1. Displacement-Time Graphs
2. Velocity-Time Graphs
3. Constant Acceleration Formulae (SUVAT)
4. Vertical Motion Under Gravity

Topics	What students need to learn:		
	Content	Guidance	
7 Kinematics	7.1	Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration.	
	7.2	Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph.	Derivation may use knowledge of Sections 7.2 and/or 7.4 Understand and use <i>suvat</i> formulae for constant acceleration in 2D.
7 Kinematics <i>continued</i>	7.3	Understand, use and derive the formulae for constant acceleration for motion in a straight line.	Derivation may use knowledge of sections 7.2 and/or 7.4 Understand and use <i>suvat</i> formulae for constant acceleration in 2D.
	7.4	Use calculus in kinematics for motion in a straight line: $v = \frac{dr}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$ $r = \int v \, dt, \quad v = \int a \, dt$	The level of calculus required will be consistent with that in Sections 7 and 8 in Paper 1.

1. Displacement-Time Graphs

Describe the motion of each object:



Velocity is the rate of change of displacement
(i.e. gradient of displacement-time graph)

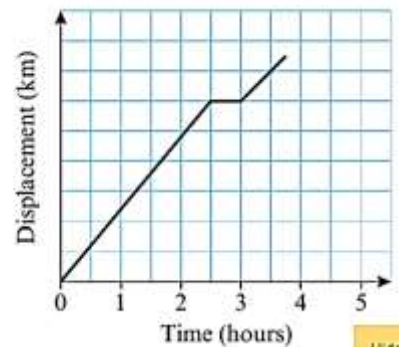
Average Velocity = _____

Average Speed = _____

Example (Exercise 9A Question 2)

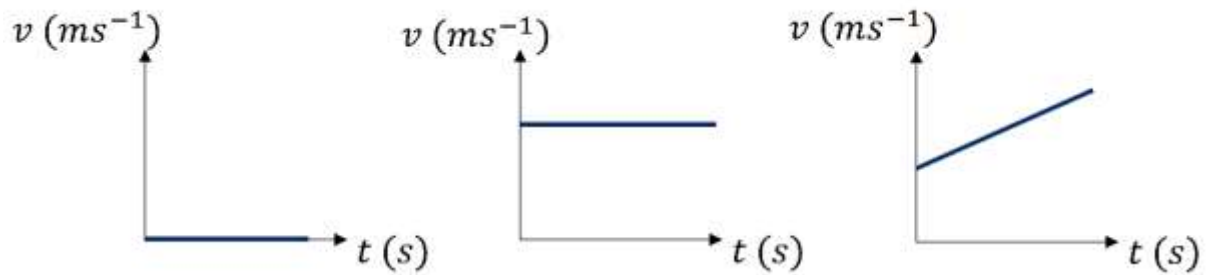
Khalid drives from his home to a hotel. He drives for $2\frac{1}{2}$ hours at an average velocity of 60 km h^{-1} . He then stops for lunch before continuing to his hotel. The diagram shows a displacement–time graph for Khalid’s journey.

- a Work out the displacement of the hotel from Khalid’s home.
- b Work out Khalid’s average velocity for his whole journey.



2. Velocity-Time Graphs

Describe the motion of each object:



Acceleration the rate of change of velocity
(i.e. gradient of velocity-time graph)

The **area** under a velocity-time graph gives the **distance travelled**.

Example

The velocity-time graph shown is for a body which starts from rest, accelerates uniformly to a velocity of 8ms^{-1} in 2 seconds, maintains that velocity for a further 5 seconds then decelerates uniformly to rest. The entire journey takes 11 seconds. Find:

- The acceleration of the body during the initial part of the motion
- The deceleration of the body during the final part of the motion
- The total distance travelled by the body



Algebraic Example

A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 ms^{-1} in T seconds. The particle then travels at a constant velocity of 8 ms^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

(a) Sketch a velocity-time graph to illustrate the motion of the particle.

Give then the total displacement of the particle is 600m.

(b) find the value of T .

Test Your Understanding (EdExcel M1 May 2013 Q5)

A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1} . The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

(a) Sketch, in the space below, a speed-time graph for the motion of the car between the two sets of traffic lights. (2)

(b) Find the value of T . (3)

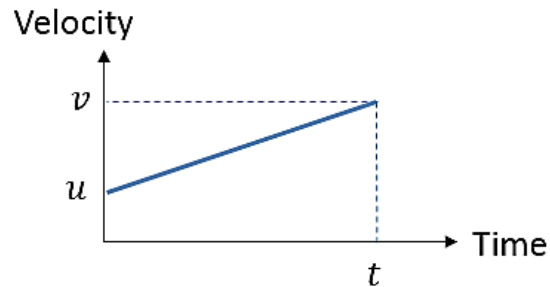
A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, $a \text{ m s}^{-2}$, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s^{-1} .

(c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A . (4)

(d) Find the value of a . (2)

3-4. Constant Acceleration Formulae (SUVAT Equations)

These formulae are used to solve problems where the object is moving in a **straight line with constant acceleration for a specific period of time**. You should memorise these and know how to derive them.



$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$v =$$

(Equation 1)

For uniform acceleration, the average velocity is the average of v and u . Using the area of the graph (which we know gives distance):

$$s =$$

(Equation 2)

Eliminating v - sub for v from equation 1 into equation 2:

$$s =$$

(Equation 3)

Eliminating t - sub for t from equation 1 into equation 2:

$$v^2 =$$

(Equation 4)

Eliminating u – sub for u from equation 1 into equation 2:

$s =$

(Equation 5)

- Work out what you know
- Work out what you need
- Choose the appropriate equation
- Diagrams help!
- Work out which direction will be positive
- Check that your units are consistent

Example

A stone slides in a straight line across a horizontal sheet of ice. It passes a point, A with velocity 14ms^{-1} and a point, B 2.5 seconds later. Assuming the deceleration is uniform and that $AB = 30\text{m}$, find:

- a) The deceleration
- b) The velocity at B
- c) How long after passing A the stone comes to rest

Example – Deceleration Leading to a Change in Direction

A particle travels with uniform deceleration 2ms^{-2} in a horizontal line. The points A and B lie on the line and $AB = 32\text{m}$. At time $t = 0$, the particle passes through A with velocity 12ms^{-1} in the direction AB. Find:

- a) The values of t when the particle is at B
- b) The velocity of the particle for each of these values of t .

Test Your Understanding (EdExcel M1 May 2013 Q4)

A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point A with speed $u \text{ m s}^{-1}$, ($u < 34$), and 10 seconds later passes a point B with speed 34 m s^{-1} . Given that $AB = 240 \text{ m}$, find

(a) the value of u , (3)

(b) the time taken for the lorry to move from A to the mid-point of AB . (6)

5. Vertical Motion Under Gravity

The downwards acceleration under gravity is $g = 9.8 \text{ ms}^{-2}$.
ALWAYS state the positive direction in your calculations.
Quote final answers to 2 or 3 s.f. – you may be penalised if you quote more.

Example

A ball is thrown vertically upwards with a velocity of 14.7 ms^{-1} from a platform 19.6 m above the ground. Find:

- a) The time taken for the ball to reach the ground
- b) The velocity of the ball when it hits the ground

Example

A ball is projected vertically upwards from ground level at a speed of 20 ms^{-1} . Determine the amount of time the ball is at least 10m above ground level.

Example – When Two Particles are in Motion

Two stones are thrown from the same point at the same time - one vertically upwards with speed 30ms^{-1} and one vertically downwards at 30ms^{-1} . Find how far apart the stones are after 3 seconds.

Test Your Understanding (EdExcel M1 May 2013 (R) Q4)

At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with speed 2 m s^{-1} from a point 50 m above the horizontal ground. The ball B is projected vertically upwards from the ground with speed 20 m s^{-1} . At time $t = T$ seconds, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T , (5)

(b) the value of h . (2)