

# Factorising expressions

## A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form  $ax^2 + bx + c$ , where  $a \neq 0$ .
- To factorise a quadratic equation find two numbers whose sum is  $b$  and whose product is  $ac$ .
- An expression in the form  $x^2 - y^2$  is called the difference of two squares. It factorises to  $(x - y)(x + y)$ .

## Examples

**Example 1** Factorise  $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$ . So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets
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**Example 2** Factorise  $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$
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**Example 3** Factorise  $x^2 + 3x - 10$

$b = 3, ac = -10$  So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	<ol style="list-style-type: none"> <li><b>1</b> Work out the two factors of <math>ac = -10</math> which add to give <math>b = 3</math> (5 and -2)</li> <li><b>2</b> Rewrite the <math>b</math> term (<math>3x</math>) using these two factors</li> <li><b>3</b> Factorise the first two terms and the last two terms</li> <li><b>4</b> <math>(x + 5)</math> is a factor of both terms</li> </ol>
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**Example 4** Factorise  $6x^2 - 11x - 10$

<p><math>b = -11, ac = -60</math></p> <p>So</p> $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> <li><b>1</b> Work out the two factors of <math>ac = -60</math> which add to give <math>b = -11</math> (<math>-15</math> and <math>4</math>)</li> <li><b>2</b> Rewrite the <math>b</math> term (<math>-11x</math>) using these two factors</li> <li><b>3</b> Factorise the first two terms and the last two terms</li> <li><b>4</b> <math>(2x - 5)</math> is a factor of both terms</li> </ol>
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**Example 5** Simplify  $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ <p>For the numerator:</p> <p><math>b = -4, ac = -21</math></p> <p>So</p> $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$ <p>For the denominator:</p> <p><math>b = 9, ac = 18</math></p> <p>So</p> $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$ <p>So</p> $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	<ol style="list-style-type: none"> <li><b>1</b> Factorise the numerator and the denominator</li> <li><b>2</b> Work out the two factors of <math>ac = -21</math> which add to give <math>b = -4</math> (<math>-7</math> and <math>3</math>)</li> <li><b>3</b> Rewrite the <math>b</math> term (<math>-4x</math>) using these two factors</li> <li><b>4</b> Factorise the first two terms and the last two terms</li> <li><b>5</b> <math>(x - 7)</math> is a factor of both terms</li> <li><b>6</b> Work out the two factors of <math>ac = 18</math> which add to give <math>b = 9</math> (<math>6</math> and <math>3</math>)</li> <li><b>7</b> Rewrite the <math>b</math> term (<math>9x</math>) using these two factors</li> <li><b>8</b> Factorise the first two terms and the last two terms</li> <li><b>9</b> <math>(x + 3)</math> is a factor of both terms</li> <li><b>10</b> <math>(x + 3)</math> is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1</li> </ol>
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## Practice

1 Factorise.

a  $6x^4y^3 - 10x^3y^4$

c  $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

b  $21a^3b^5 + 35a^5b^2$

2 Factorise

a  $x^2 + 7x + 12$

c  $x^2 - 11x + 30$

e  $x^2 - 7x - 18$

g  $x^2 - 3x - 40$

b  $x^2 + 5x - 14$

d  $x^2 - 5x - 24$

f  $x^2 + x - 20$

h  $x^2 + 3x - 28$

3 Factorise

a  $36x^2 - 49y^2$

c  $18a^2 - 200b^2c^2$

b  $4x^2 - 81y^2$

4 Factorise

a  $2x^2 + x - 3$

c  $2x^2 + 7x + 3$

e  $10x^2 + 21x + 9$

b  $6x^2 + 17x + 5$

d  $9x^2 - 15x + 4$

f  $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a  $\frac{2x^2 + 4x}{x^2 - x}$

c  $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e  $\frac{x^2 - x - 12}{x^2 - 4x}$

b  $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d  $\frac{x^2 - 5x}{x^2 - 25}$

f  $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a  $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c  $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b  $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d  $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

### Hint

Take the highest common factor outside the bracket.

## Extend

7 Simplify  $\sqrt{x^2 + 10x + 25}$

8 Simplify  $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

**Answers**

- 1 a  $2x^3y^3(3x - 5y)$   
c  $5x^2y^2(5 - 2x + 3y)$
- 2 a  $(x + 3)(x + 4)$   
c  $(x - 5)(x - 6)$   
e  $(x - 9)(x + 2)$   
g  $(x - 8)(x + 5)$
- 3 a  $(6x - 7y)(6x + 7y)$   
c  $2(3a - 10bc)(3a + 10bc)$
- 4 a  $(x - 1)(2x + 3)$   
c  $(2x + 1)(x + 3)$   
e  $(5x + 3)(2x + 3)$
- 5 a  $\frac{2(x+2)}{x-1}$   
c  $\frac{x+2}{x}$   
e  $\frac{x+3}{x}$
- 6 a  $\frac{3x+4}{x+7}$   
c  $\frac{2-5x}{2x-3}$
- 7  $(x + 5)$
- 8  $\frac{4(x+2)}{x-2}$
- b  $7a^3b^2(3b^3 + 5a^2)$
- b  $(x + 7)(x - 2)$   
d  $(x - 8)(x + 3)$   
f  $(x + 5)(x - 4)$   
h  $(x + 7)(x - 4)$
- b  $(2x - 9y)(2x + 9y)$
- b  $(3x + 1)(2x + 5)$   
d  $(3x - 1)(3x - 4)$   
f  $2(3x - 2)(2x - 5)$
- b  $\frac{x}{x-1}$   
d  $\frac{x}{x+5}$   
f  $\frac{x}{x-5}$
- b  $\frac{2x+3}{3x-2}$   
d  $\frac{3x+1}{x+4}$