Chapter 10 - Mechanics

Forces and Motion

Chapter Overview

1. Force Diagrams
2. Forces as Vectors
3. Forces and Acceleration
4. Motion in 2 Dimensions
5. Connected Particles
6. Pulleys

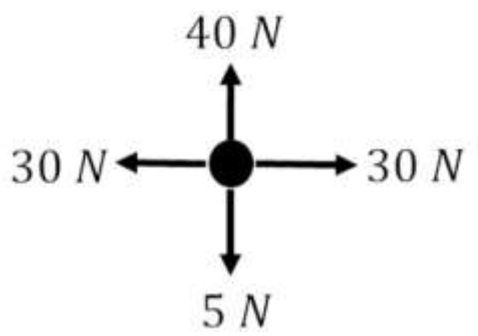
Topics	What students need to learn:		
	Content		Guidance
8 **Forces and Newton's laws**	8.1	Understand the concept of a force; understand and use Newton's first law.	Normal reaction, tension, thrust or compression, resistance.
	8.2	Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors).	Problems will involve motion in a straight line with constant acceleration in scalar form, where the forces act either parallel or perpendicular to the motion. Resolving forces is not required. Problems will involve motion in a straight line with constant acceleration in vector form, where the forces are given in $\mathbf{i}-\mathbf{j}$ form or as column vectors.
	8.3	Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy.	The default value of g will be 9.8 m s^{-2} but some questions may specify another value, e.g. $g = 10\text{ m s}^{-2}$ The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location.
	8.4	Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line; application to problems involving smooth pulleys and connected particles.	Equilibrium problems will not require forces to be resolved (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors). For pulley problems, the strings will either be horizontal or vertical. Connected particle problems could include problems with particles in contact, e.g. lift problems.

1. <u>Force Diagrams</u>

Recall Newton's laws of motion:

1. An object will remain at rest or continue to move **in a straight line at a constant speed** unless it is acted upon by a **resultant force**
2. The force needed to accelerate a particle is equal to the product of its mass and the acceleration of the particle: **F = ma**
3. Every action has an equal and opposite reaction

When drawing a force diagram, make sure you include **<u>all</u>** of the forces which might be acting on an object (see Chapter 8). Consider the forces acting on each object, one at a time. You can draw the resultant force and describe the motion of the object.



$R(\uparrow)$:

$R(\rightarrow)$:

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2. Forces as Vectors

Forces have direction, and therefore we can naturally write them as vectors, either in $\boldsymbol{i}$-$\boldsymbol{j}$ notation or as column vectors.

Add the vectors of two or more forces to find the resultant force.

Example

The forces (3***i*** - 4***j***), (2***i*** + 5***j***) and (a***i*** + b***j***) act on a particle in equilibrium. Find the values of a and b.

If the particle is in equilibrium, what is the value of the resultant force?

We can use Pythagoras and trignometry to find the magnitude and bearing of a force when it is in vector form.

Example

The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.

(b) Work out the magnitude and bearing of the resultant force.

Test Your Understanding *(EdExcel M1 Jan 2012 Q3)*

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\text{ N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\text{ N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q.

(3)

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$.
Find

(b) the magnitude of $\mathbf{R}$,

(2)

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$.

(3)

Test Your Understanding *(EdExcel M1 May 2009 Q2)*

A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

(a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$.

(2)

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

(b) find the value of p.

(4)

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3. Forces and Acceleration

Newton's 2nd Law of Motion: $F = ma$
(where the force F and acceleration a are in the same direction)

- Force is measured in Newtons (N)
- Mass is measured in kg
- Acceleration is measured in ms^{-2}

If F = ma
N = $kgms^{-2}$

Examples

1. A car of 2000kg has a driving force of 800N and forces of 200N resisting its motion. Determine its acceleration.

2. A child has a mass of 50kg. What is the gravitational force acting on the child? (i.e. its weight)

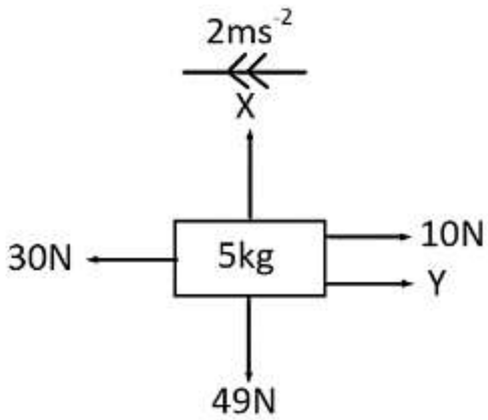
Combining F = ma with SUVAT equations

We can use SUVAT equations and Newton 1 and 2 to solve problems. We **resolve** forces which are parallel in one or more directions to do this.
Forces acting in a **perpendicular** direction do not affect the motion of a body.

NB: Remember SUVAT is for <u>constant acceleration only</u>.

Example

The forces acting on a body cause it to accelerate as indicated.
a) Find the values of X and Y
b) Find the distance travelled in the first 4 seconds if the object starts at rest.



(Indicate which direction is positive vertically and horizontally)

Forces Acting Under Gravity

Acceleration due to gravity is $g = 9.8$ ms^{-2}

Example

A lift of mass 600kg is raised or lowered by means of a cable attached to its top. When carrying passengers whose total mass is 400kg, the lift accelerates uniformly from rest to 2ms^{-1} over a distance of 5m. Find:

a) The magnitude of the acceleration

b) The tension in the cable if the motion takes place vertically upwards

c) The tension in the cable if the motion takes place vertically downwards

Test Your Understanding *(EdExcel M1 May 2012 Q5 abridged)*

A particle P is projected vertically upwards from a point A with speed u m s^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 m s^{-1}.

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P.

(c) Find the vertical distance that P sinks into the ground before coming to rest.

(4)

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4. Motion in Two Dimensions

Force and Acceleration can be represented as both scalars and vectors. Therefore Newton's 2nd law can be used in vector form too.

This naturally means that $\boldsymbol{F} = m\boldsymbol{a}$ works with vectors too.

Example

Forces $\boldsymbol{F}_1$ $(4\boldsymbol{i} - 7\boldsymbol{j})$, and $\boldsymbol{F}_2$ $(-6\boldsymbol{i} + 2\boldsymbol{j})$ and $\boldsymbol{F}_3$ $(4\boldsymbol{j})$ act on a particle of mass 2kg. Find the acceleration of the particle. Find also the magnitude and the bearing of the acceleration.

Example – Using SUVAT Equations

A constant force $\boldsymbol{F}$ N acts on a particle of mass 5kg for 8 seconds. The particle is initially at rest and 8 seconds later it has velocity $(12\boldsymbol{i} - 5\boldsymbol{j})$ ms^{-1}. Find $\boldsymbol{F}$.

Test Your Understanding *(Textbook)*

A boat is modelled as a particle of mass 60 kg being acted on by three forces.

$$\boldsymbol{F}_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix} N, \quad \boldsymbol{F}_2 = \begin{pmatrix} 10p \\ 20q \end{pmatrix} N, \quad \boldsymbol{F}_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$ ms^{-2}, find the values of p and q.

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5. Connected Particles

When we have multiple connected objects moving in the same straight line, **they can be considered either as two separate particles, or as a single particle**, but **all forces** acting on the particle must be considered.

What assumptions are made?

Example *(EdExcel M1 June 2009 Q6)*

A car of mass 800 kg pulls a trailer of mass 200 kg along a straight horizontal road using a light towbar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 400 N and 200 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N. Find

(a) the acceleration of the car and trailer,

(3)

(b) the magnitude of the tension in the towbar.

(3)

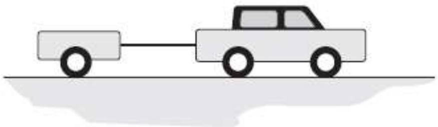
The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate. Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N,

(c) find the value of F.

(7)

Test Your Understanding

4. A car of mass 900 kg is towing a trailer of mass 100 kg along a horizontal road. There are resistance forces of 60 N and 20 N acting on the car and the trailer respectively.

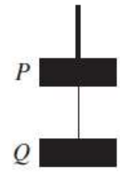


(a) The engine applies a driving force of 480 N. Calculate
   (i) the acceleration of the car and trailer
   (ii) the tension in the towbar. (4)

(b) The brakes are now applied, bringing the car to a halt from a speed of 10 ms^{-1} in a distance of 50 m. Find
   (i) the magnitude of the braking force
   (ii) the force in the towbar. (6)

Vertical Example

A brick P of mass 4 kg is suspended by a vertical, light inextensible string. Another brick Q of mass 6 kg is suspended from P by another light inextensible string, as shown in the diagram. The bricks start from rest and are then raised 2 m in 4 seconds. Find the tension in each string.



Example – Using Newton's 3rd Law for Stacked Objects

Newton's 3rd Law: For every action there is an equal and opposite reaction

Therefore when two bodies A and B are in contact, if body A exerts a force on body B, then body B exerts a force on body A that is equal in magnitude and acts in the opposite direction.

[Textbook] A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400g and the mass of B is 600g. A rests on top of B, as shown in the diagram.

The scale-pan is raised vertically, using the string, with acceleration 0.5 ms^{-2}.

(a) Find the tension in the string.
(b) Find the force exerted on mass B by mass A.
(c) Find the force exerted on mass B by the scale-pan.

Test Your Understanding – Motion of a Lift *(EdExcel M1 May 2013 Q2)*

A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 m s^{-2}. By modelling the cable as being light and inextensible, find

(a) the tension in the cable,

(3)

(b) the magnitude of the force exerted on the woman by the floor of the lift.

(3)

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6. <u>Pulleys</u>

A pulley is a wheel on which a rope/string/cable passes.

What modelling assumptions are made?

Example

Particles of mass 4kg and 2kg are connected by a light string passing over a smooth, fixed pulley. The particles hang freely and are released from rest.

i) Find the acceleration of the two particles and the tension in the string. Let the acceleration be a and the tension in the string be T

ii) Find the force exerted on the pulley by the string

Example – Horizontal and Vertical String

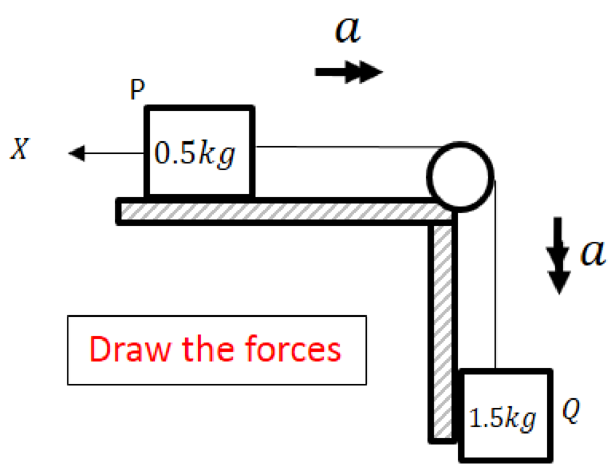
(Take g = 10ms^{-2} in this question)

The diagram shows a particle, P, of mass 0.5kg on a smooth horizontal table. P is connected to another particle, Q, of mass 1.5kg, by a taut, light, inextensible string which passes over a small, fixed, smooth pulley at the edge of the table, Q hanging vertically below the pulley.

A horizontal force of magnitude *X*N acts on P as shown.

a) Given the system is in equilibrium, find *X*

b) Given that *X* = 12, find the distance travelled by Q in the first two seconds of its motion, following the release of the system from rest. You may assume that P does not reach the pulley in this time.



<u>Test Your Understanding</u> *(EdExcel M1 Jan 2010 Q6)*

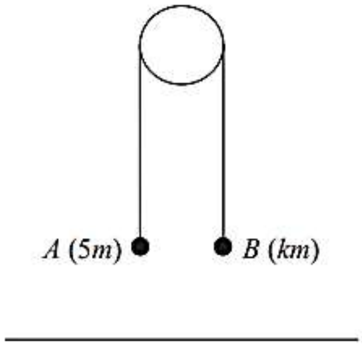


Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.
(3)

(b) Find the value of k.
(3)

(c) State how you have used the information that the pulley is smooth.
(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane.
(7)

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Mixed Exercise 10 Page 177