MechYr2 Chapter 4 :: Moments

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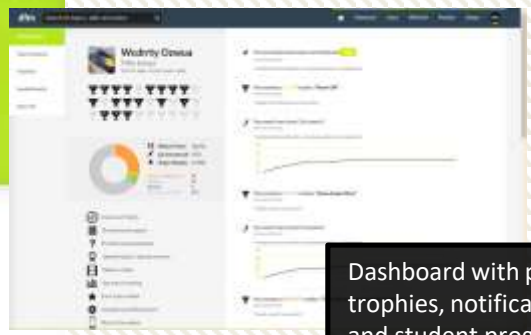
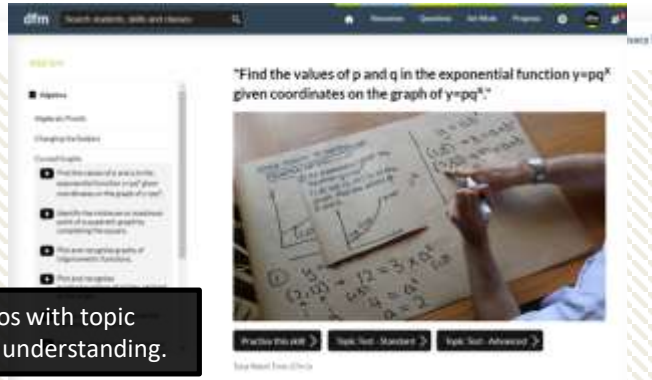
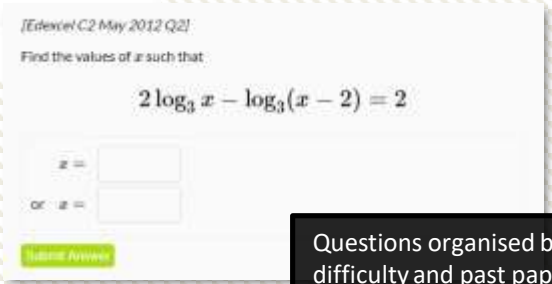


Dashboard with points, trophies, notifications and student progress.



Teaching videos with topic tests to check understanding.



Questions organised by topic, difficulty and past paper.

Motivating problem

This… is a door.

Why do you think the handle is put on the other side of the door from the hinge?

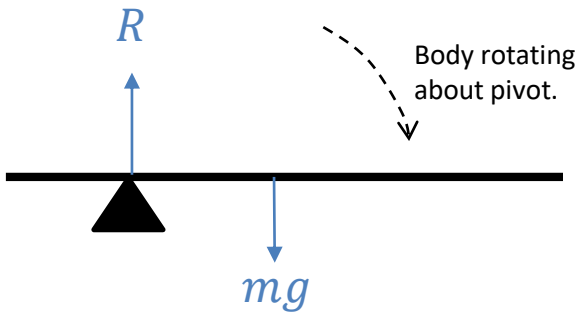
?

If I double the distance of my finger from the hinge, what happens to the force required to keep the door open?

?

Rigid Bodies and Overview

We previously dealt with particles, where each object was modelled just as a single point, and considered forces acting on each point separately.



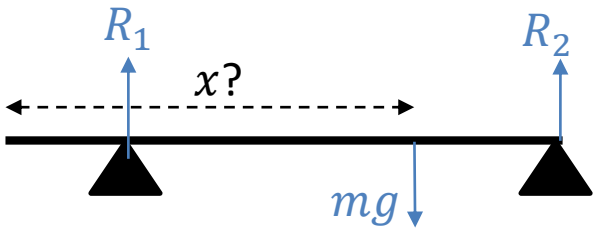
In this chapter we consider rigid bodies (in this case **rods**), which takes into account the size of the object. This means we can consider other properties, e.g. **rotation** of the body.

1:: Moments in equilibrium

Clockwise moment = Anticlockwise moment

2:: Centre of Mass

For a 'non-uniform' rod we can't model its weight as acting at the centre.



3:: On the point of Tilting

"A non uniform wooden plank of mass M kg rests horizontally on supports at A and B, as shown. When a bucket of water of mass 18kg is placed at point C, the plank is in equilibrium, and is **on the point of tilting** about B. Find the value of M and the magnitude of the reaction at B."

Moments

The ‘moment’ of a force:

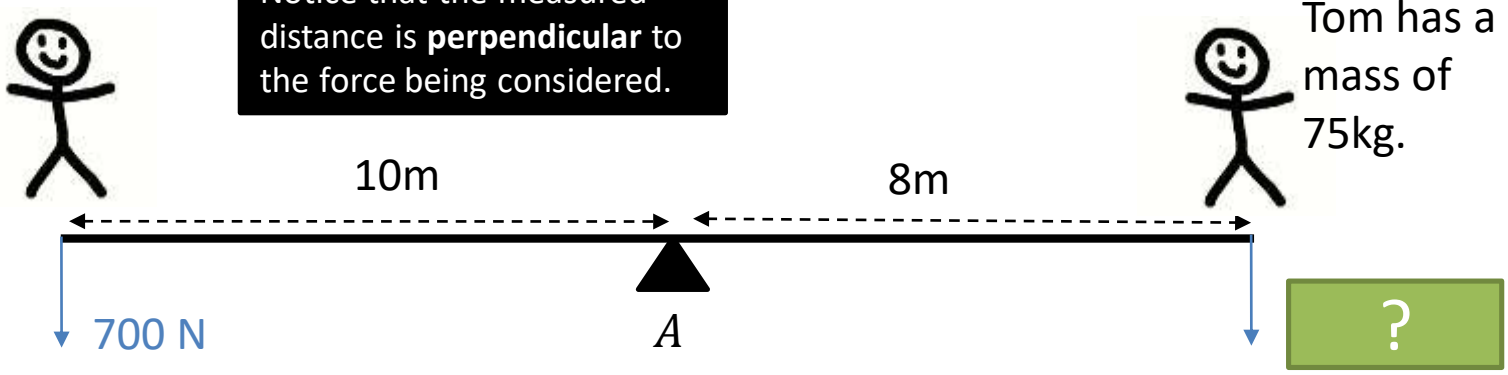
?

$$moment\ of\ force = force \times distance$$

about a point

perpendicular distance

Notice that the measured distance is **perpendicular** to the force being considered.



Tom has a mass of 75kg.

Moment about A

$=$?

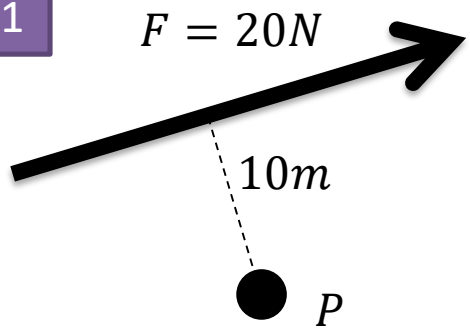
Moment about A

$=$?

The anticlockwise moment is greater, so the seesaw will tilt in an anticlockwise direction.

Quickfire Examples

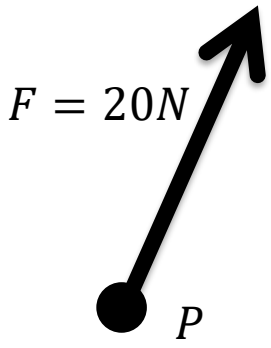
1



Moment of force F about point P

$=$?

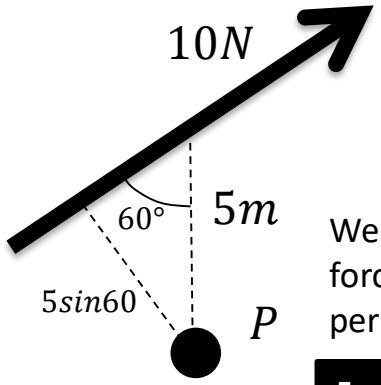
2



Moment of force F about point P

?

3



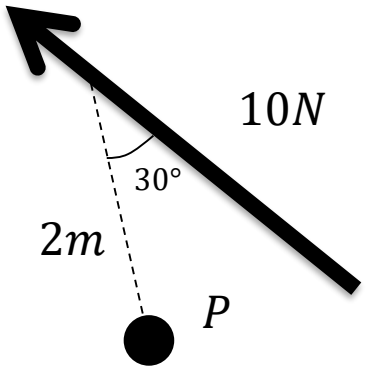
Moment of force F about point P

$=$?

We see what force is acting perpendicularly.

Fro Tip: Recall that if the hypotenuse of a right-angled triangle is x then the side opposite the angle is of length $x \sin\theta$ and the adjacent to it $x \cos\theta$. You should be able to determine these lengths instantly without piddling about with soh-cah-toa each time.

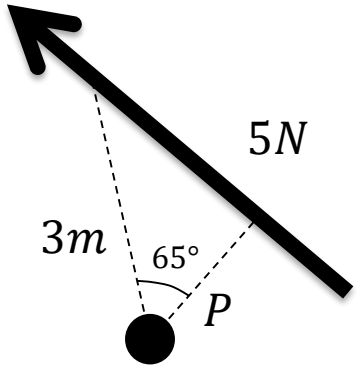
4



Moment:

?

5

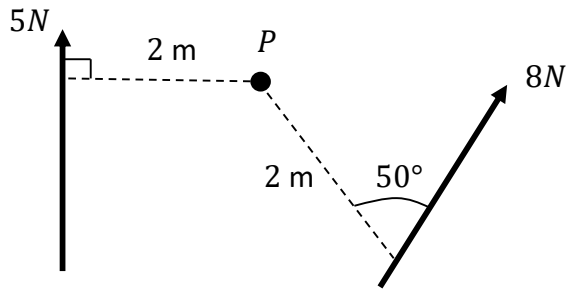


Moment:

?

Test Your Understanding

[Textbook] The diagram shows two forces acting on a lamina. Find the moment of each of the forces about the point P.



Terminology: A *lamina* is a 2D object whose thickness can be ignored.

Moment of 5N force:

= ?

Moment of 8N force:

= ?

Exercise 4A

Pearson Stats/Mechanics Year 2
Page 72

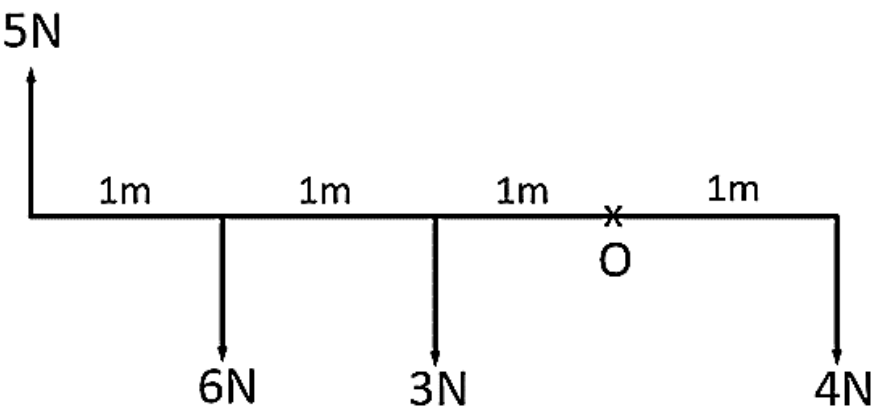
(Classes in a rush may wish to skip this exercise)

Resultant moments

If we have multiple coplanar forces, we can also find the overall moment by adding them – just treat one of the directions (clockwise or anticlockwise) as negative. This is similar in Year 1 to finding the **resultant force**.

Find the sum of moments about the point O, of the forces acting in each diagram:

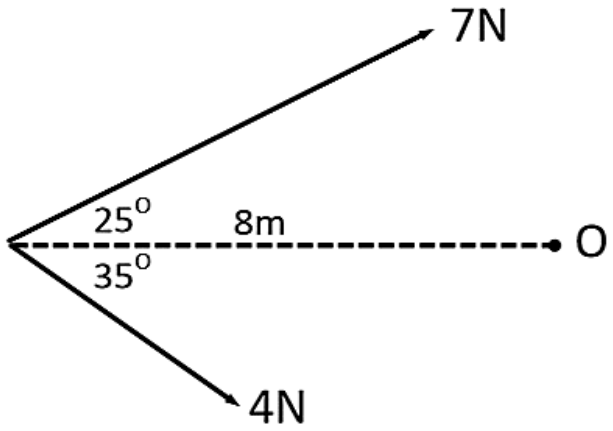
a)



Resultant moment (about O)

?

b)

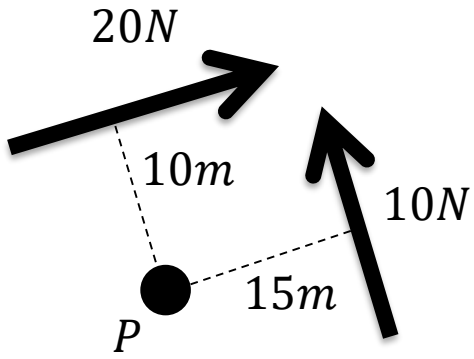


Resultant moment (about O)

?

Quickfire Questions

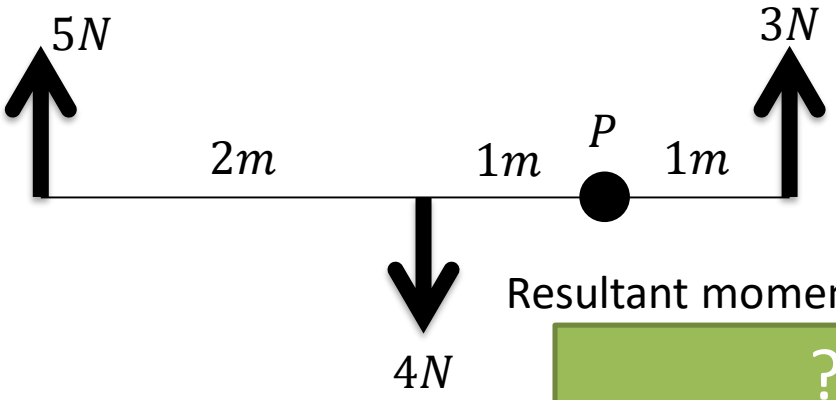
1



Resultant moment (about P)

?

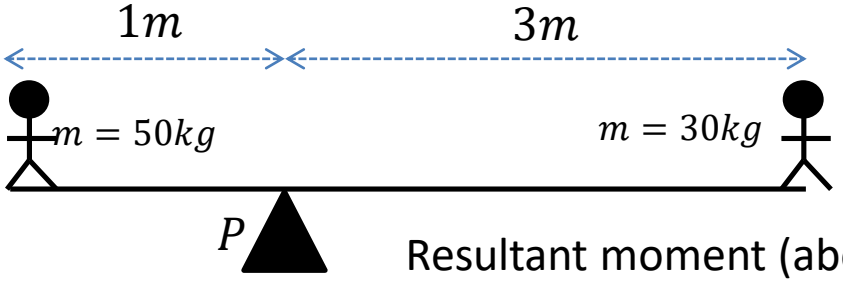
2



Resultant moment (about P):

?

3

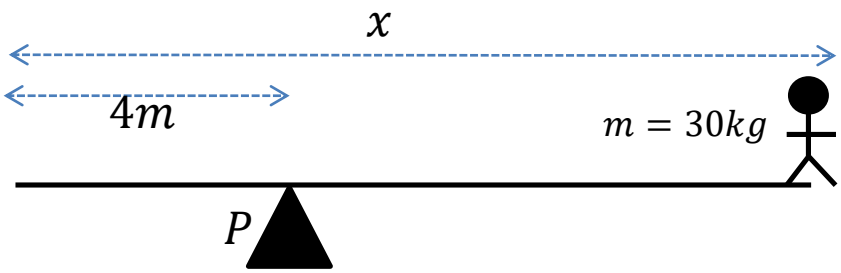


(Rod is light)

Resultant moment (about P):

?

4



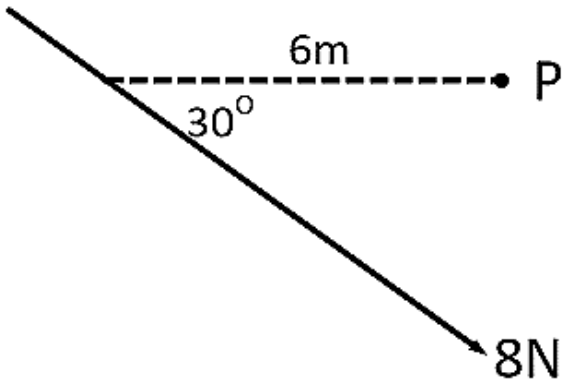
Resultant moment (about P):

?

Resolving a Force into its Parallel and Perpendicular Components to find its Moment

This can be a useful technique for harder problems. We can use trig to find the perpendicular distance of a force from a point of rotation, but we can also split the force into its parallel and perpendicular components.

Find the moment of the force about point P.



? Resolve the 8N force into components acting parallel and perpendicular to the 6m distance

?

Exercise 4B

Pearson Stats/Mechanics Year 2
Page 74-76

This whole chapter in a nutshell…

If a rigid body is in **equilibrium** then:

a The resultant force in any direction is 0.

b The resultant moment about any point is 0.

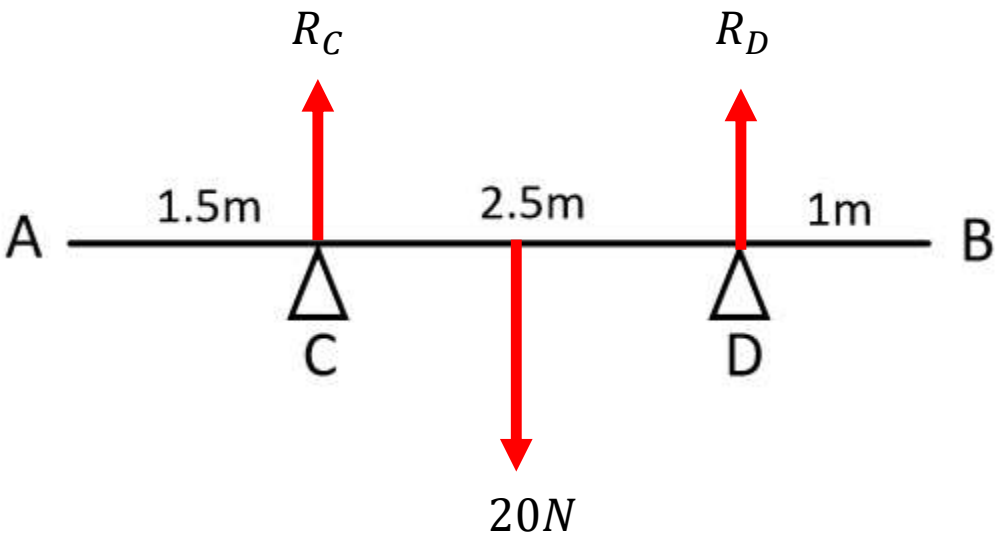
i.e. Forces up = forces down, as per Year 1

In other words, clockwise moments = anticlockwise moments

You will typically use **both** these properties to solve exam questions.

Example

AB is a uniform rod of length 5m and weight 20N. AB is resting in a horizontal position on supports at C and D. Find the magnitude of the reactions at C and D.



If a rod is uniform its centre of mass is at its centre, so we model its weight as acting at that point.

If the system is in equilibrium,
forces up = forces down
clockwise moments = anticlockwise moments

Tip: First draw on forces. Click to see forces

Method 1:

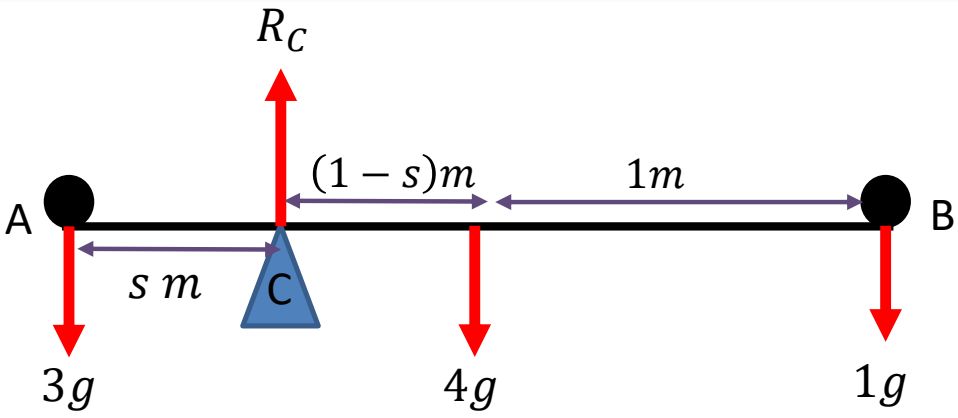
?

Method 2:

?

Example

A uniform beam AB, of length 2m and mass 4kg, has a mass of 3kg attached to one end and a mass of 1kg attached at the other end. Find the position of the support C, if the beam rests in a horizontal position.



Remember to include all forces on your force diagram.

There are two ways to solve this problem:
1) Take moments about C
2) Resolve forces to find the reaction at C, then take moments about one end

Tip: First draw on forces. Click to see forces

Tip: Draw on distances Click to see distances

Method 1:

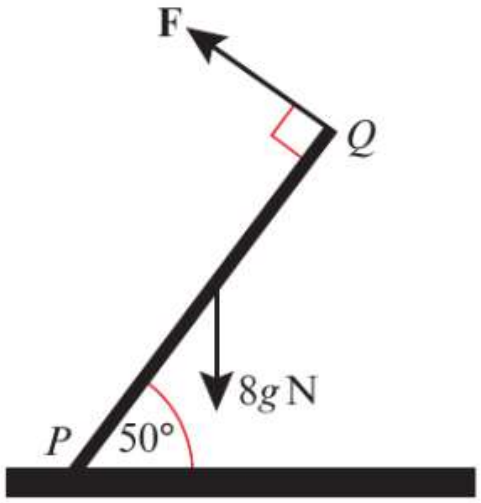
?

Method 2:

?

Test Your Understanding

A uniform rod PQ is hinged at the point P, and is held in equilibrium at an angle of $50°$ to the horizontal by a force of magnitude **F** acting perpendicular to the rod at Q. Given that the rod has a length of 3 m and a mass of 8 kg, find the value of **F**.



Method 1 – Resolve the weight perpendicular to the rod:

?

Method 2 – Work out the perpendicular distance from the weight to P:

?

Test Your Understanding

Edexcel M1(Old) May 2013(R) Q8

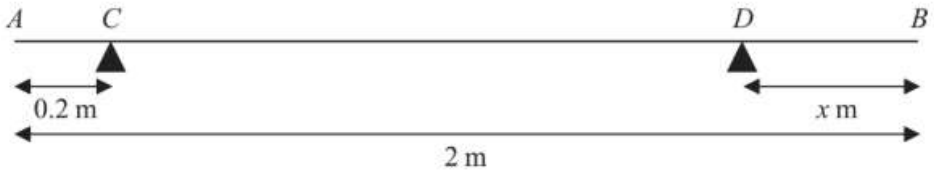


Figure 5

A uniform rod AB has length 2 m and mass 50 kg. The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D, where $AC = 0.2$ metres and $DB = x$ metres, as shown in Figure 5. Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C,

(a) find the value of x.

(6)

The support at D is now moved to the point E on the rod, where $EB = 0.4$ metres. A particle of mass m kg is placed on the rod at B, and the rod remains in equilibrium in a horizontal position. Given that the magnitude of the reaction on the rod at E is four times the magnitude of the reaction on the rod at C,

(b) find the value of m.

(7)

a

?

b

?

Exercise 4C

Pearson Stats/Mechanics Year 2
Page 78-80

Centres of Mass

So far we have assumed that the rod is uniform, that is, its mass is equally distributed across the rod, such that the centre of mass is the centre.
But this may not be the case, and for **non-uniform** rods we may wish to find where the centre of mass lies, or we will be told where it lies.

Sam and Tasmin are sitting on a non-uniform plank AB, of mass 28kg and length 5m. The plank is pivoted at M, the midpoint of AB. The centre of mass of AB is at C, where AC is 2.2m. Sam has mass 40kg. Tasmin has mass 35kg and sits at A. Where must Sam sit for the plank to be horizontal?

? Diagram

? Working

Example

Two sand bags of masses 7kg and 3kg are placed on the ends of a non-uniform rod PQ, of mass 8kg and length 4m, with the 7kg mass placed at P.
The rod rests in equilibrium on the edge of a smooth table, with half of the rod lying on the table's surface. Find the distance of the centre of mass from the edge of the table and the reaction force of the table on the rod.

? Diagram

If the rod were to tilt, where would it pivot? This will be where the reaction of the table on the rod is positioned.

? Working

Test Your Understanding

Edexcel M1(Old) May 2012 Q2

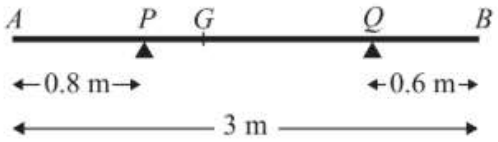


Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod,

(3)

(b) the distance AG.

(4)

? Diagram

(a)

?

(b)

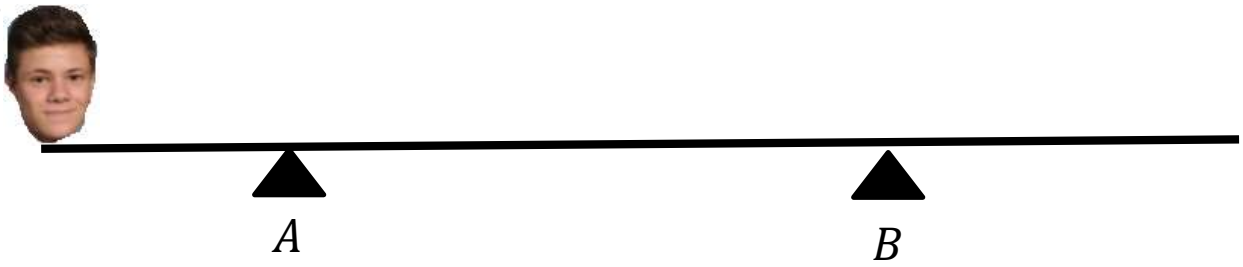
?

Exercise 4D

Pearson Stats/Mechanics Year 2
Page 81-83

Tilting

Lewis

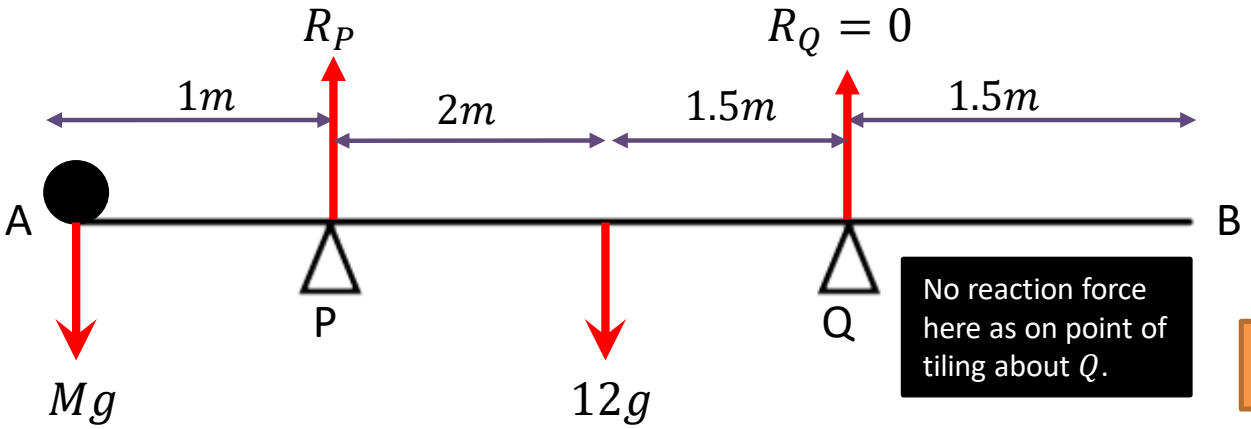


Lewis gradually increases the weight he is applying at one end of a beam, until the rod is on the verge of tilting about the support A. What can we say about the forces at B?

?

When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or tension in any other wire/string) is zero.

Example



A uniform beam AB, of mass 12kg and length 6m rests on two pivots at P and Q, where AP = 1m and QB = 1.5m.
A particle of *M* kg is placed at A and the beam is about to tilt about the pivot at P. Find the mass of the particle and the reaction force at P.

?

Test Your Understanding - Suspended System

[Textbook] A non-uniform rod AB, of length 10 m and weight 40 N, is suspended from a pair of light cables attached to C and D where $AC = 3$ m and $BD = 2$ m.
When a weight of 25 N is hung from A the rod is on the point of rotating.
Find the distance of the centre of mass of the rod from A.

? Diagram

? Working

Test Your Understanding

Edexcel M1(Old) May 2013 Q6

A beam AB has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points P and Q, where $AP = 2$ m and $QB = 3$ m. When a child of mass 50 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P. When the same child of mass 50 kg stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from A.

(8)

When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

(6)

a

(i)

(ii)

?

b

?

Exercise 4E

Pearson Stats/Mechanics Year 2
Page 84-85