Lower 6 Chapter 9

Trig Ratios

Chapter Overview

1. Sine/ Cosine Rule
2. Areas of Triangles
3. Trig Graphs
4. Proof of Sine/ Cosine Rule

5 **Trigonometry**	5.1	**Understand and use the definitions of sine, cosine and tangent for all arguments;** **the sine and cosine rules;** **the area of a triangle in the form** $\frac{1}{2}ab\sin C$	**Use of x and y coordinates of points on the unit circle to give cosine and sine respectively,** **including the ambiguous case of the sine rule.**

5.3	**Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.**	**Knowledge of graphs of curves with equations such as $y = \sin x$, $y = \cos(x + 30°)$, $y = \tan 2x$ is expected.**

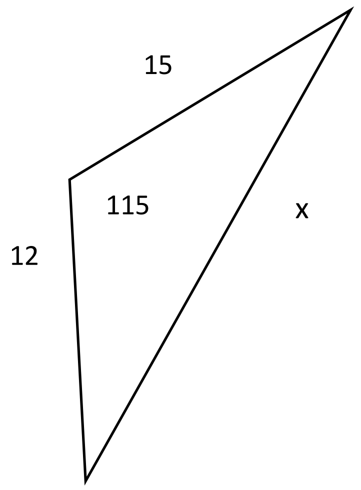
Sine and Cosine Rule

The Cosine Rule

You have	You want	Use
#1: Two angle-side opposite pairs	Missing angle or side in one pair	Sine rule
#2 Two sides known and a missing side opposite a known angle	Remaining side	Cosine rule
#3 All three sides	An angle	Cosine rule
#4 Two sides known and a missing side not opposite known angle	Remaining side	Sine rule twice

Examples:

1.



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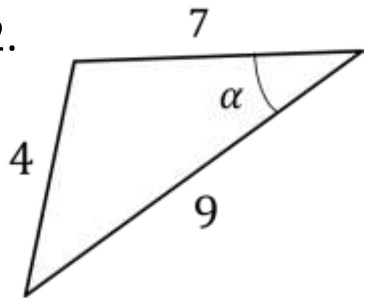

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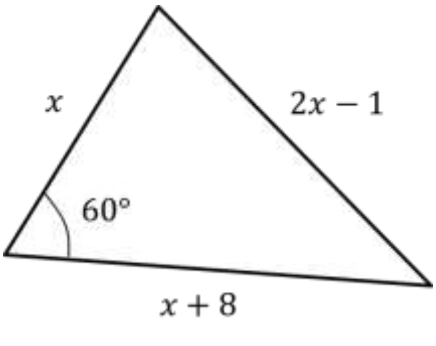
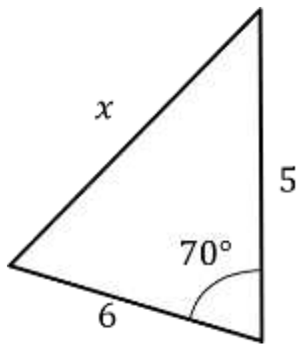


4. Coastguard station B is 8 km, on a bearing of $060°$, from coastguard station A. A ship C is 4.8 km on a bearing of $018°$, away from A. Calculate how far C is from B.

Test Your understanding

1.



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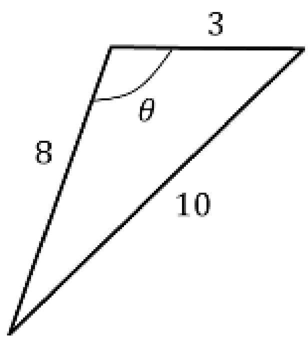

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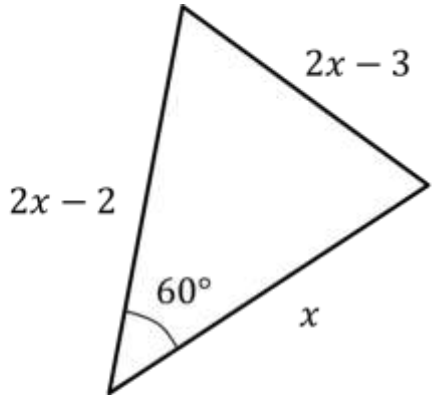
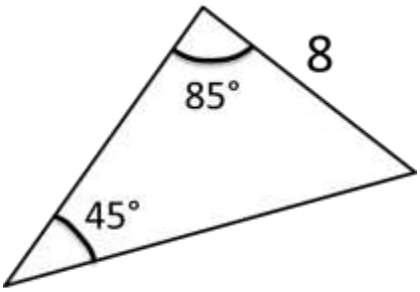


Ex 9A Pg 177

The Sine Rule

Examples:

1.



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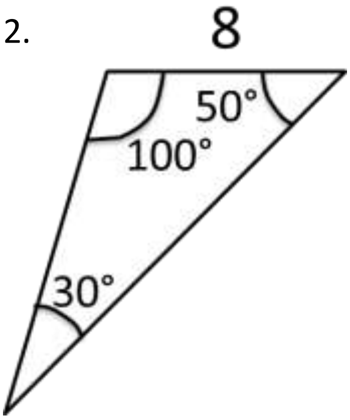

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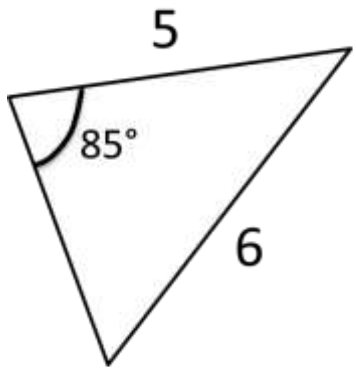

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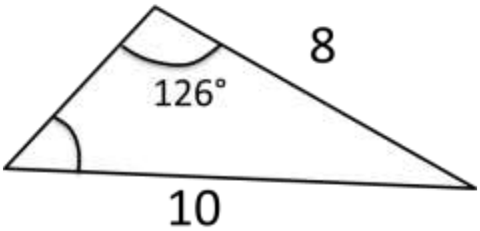
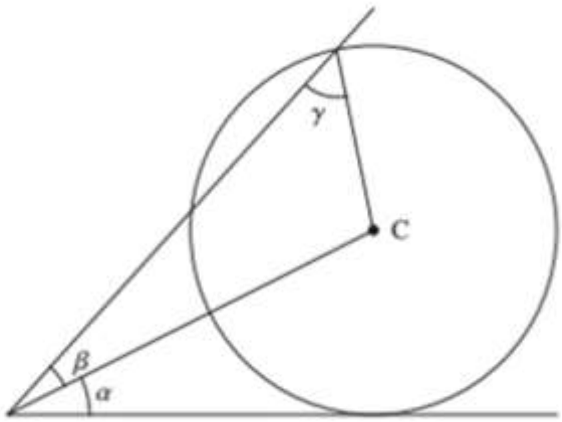


Extension

[MAT 2011 1E]

The circle in the diagram has centre C. Three angles α, β, γ are also indicated.



The angles α, β, γ are related by the equation:

A) $\cos\alpha = \sin(\beta + \gamma)$

B) $\sin\beta = \sin\alpha \sin\gamma$

C) $\sin\beta(1 - \cos\alpha) = \sin\gamma$

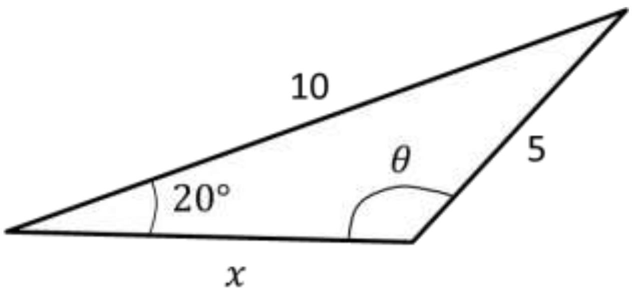
D) $\sin(\alpha + \beta) = \cos\gamma \sin\alpha$

Ex 9B Pg 181

The Ambiguous Case

Example:

Given that the angle θ is obtuse, determine θ and hence determine the length of x.

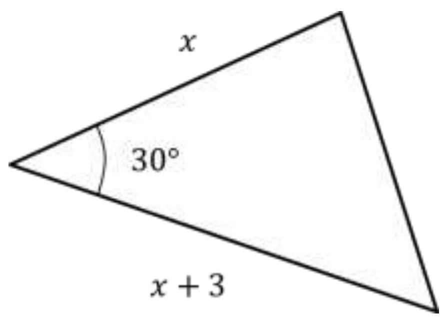


Ex 9C Pg 184

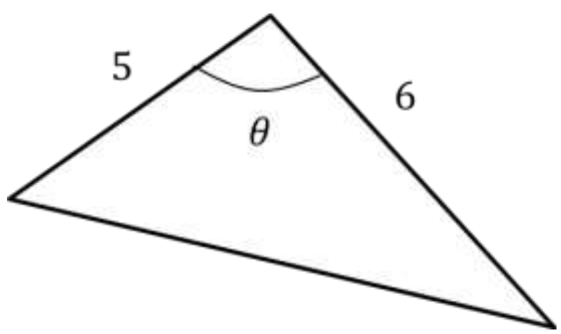
Area of Non Right-Angled Triangles

Test your understanding:

1. The area of this triangle is 10. Determine x.



2. The area of this triangle is also 10. If θ is obtuse, determine θ.



Ex 9D Pg 186

Problem solving with sin/cos rule

Example

The diagram shows the locations of four mobile phone masts in a field, $BC = 75\ m.\ CD = 80m$, angle $BCD = 55°$ and angle $ADC = 140°$.

In order that the masts do not interfere with each other, they must be at least 70m apart.

Given that A is the minimum distance from D, find:

a) The distance A is from B

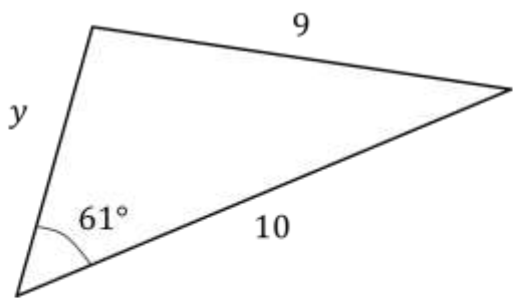
b) The angle BAD

c) The area enclosed by the four masts.

Using the sine rule twice:

Test your understanding

1.



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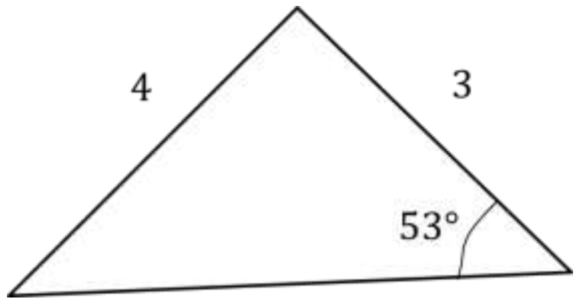


Extension

1. [AEA 2009 Q5a] The sides of the triangle ABC have lengths $BC = a,\ AC = b$ and $AB = c$, where $a < b < c$. The sizes of the angles $A,\ B$ and C form an arithmetic sequence.

   (i) Show that the area of triangle ABC is $ac\frac{\sqrt{3}}{4}$.

Given that $a = 2$ and $\sin A = \frac{\sqrt{15}}{5}$, find

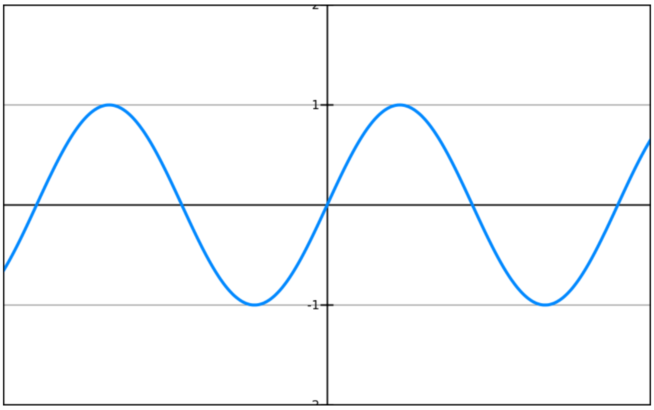
(ii) the value of b,

(iii) the value of c.

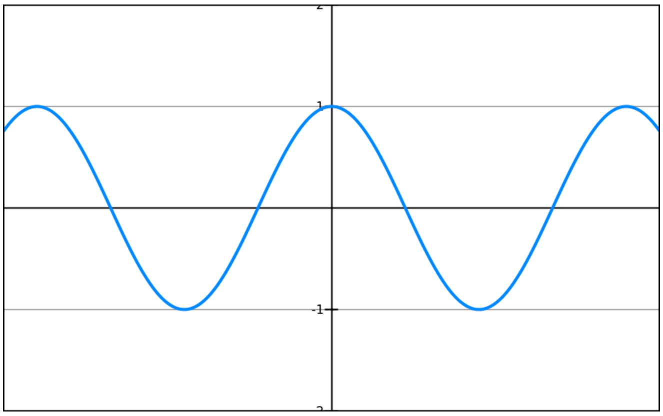
Ex 9E Pg 189

<u>Trig Graphs</u>

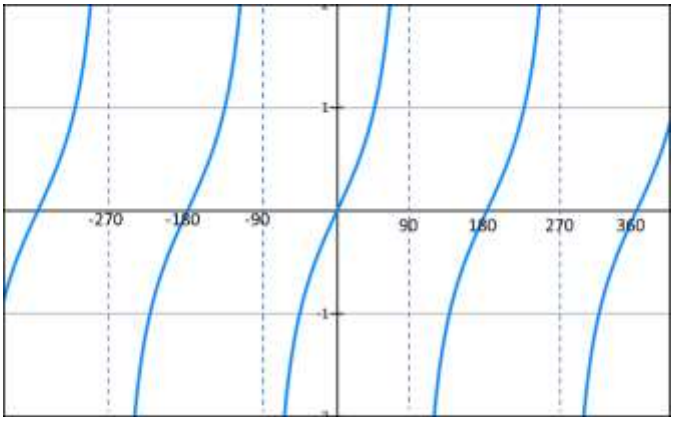
Y = sin x



Y = cos x



Y = tan x



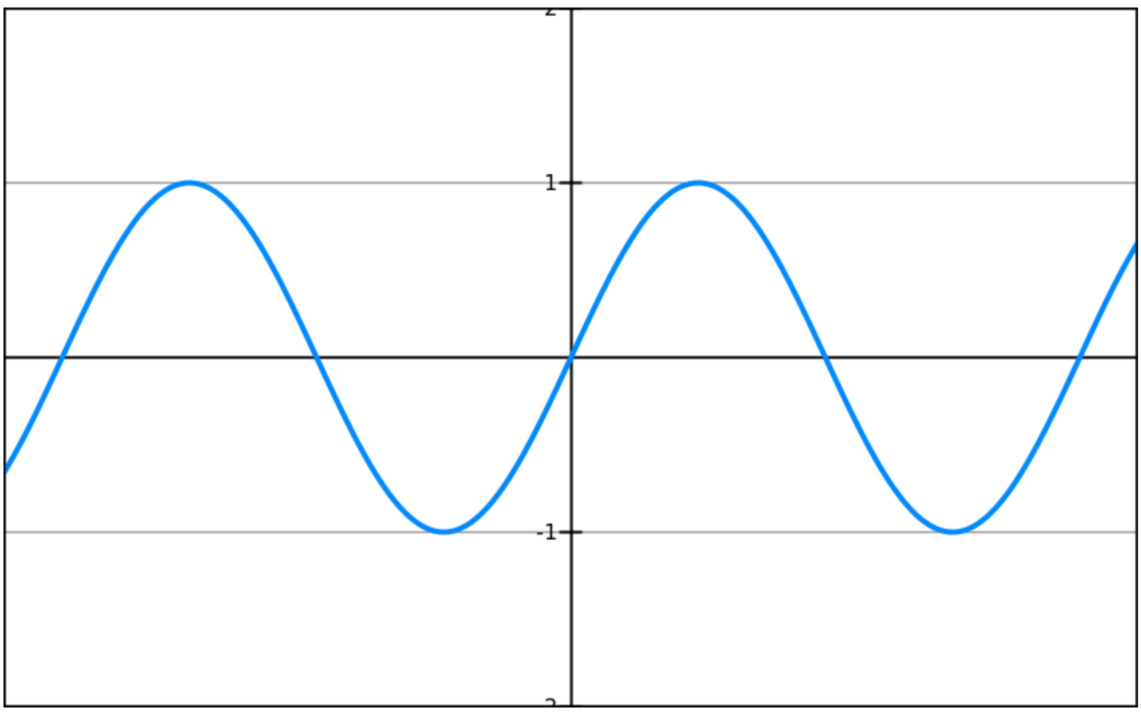
Using trig graphs

Suppose we know that sin(30) = 0.5. By thinking about symmetry in the graph, how could we work out:

Sin(150)

Sin(-30)

Sin(210)

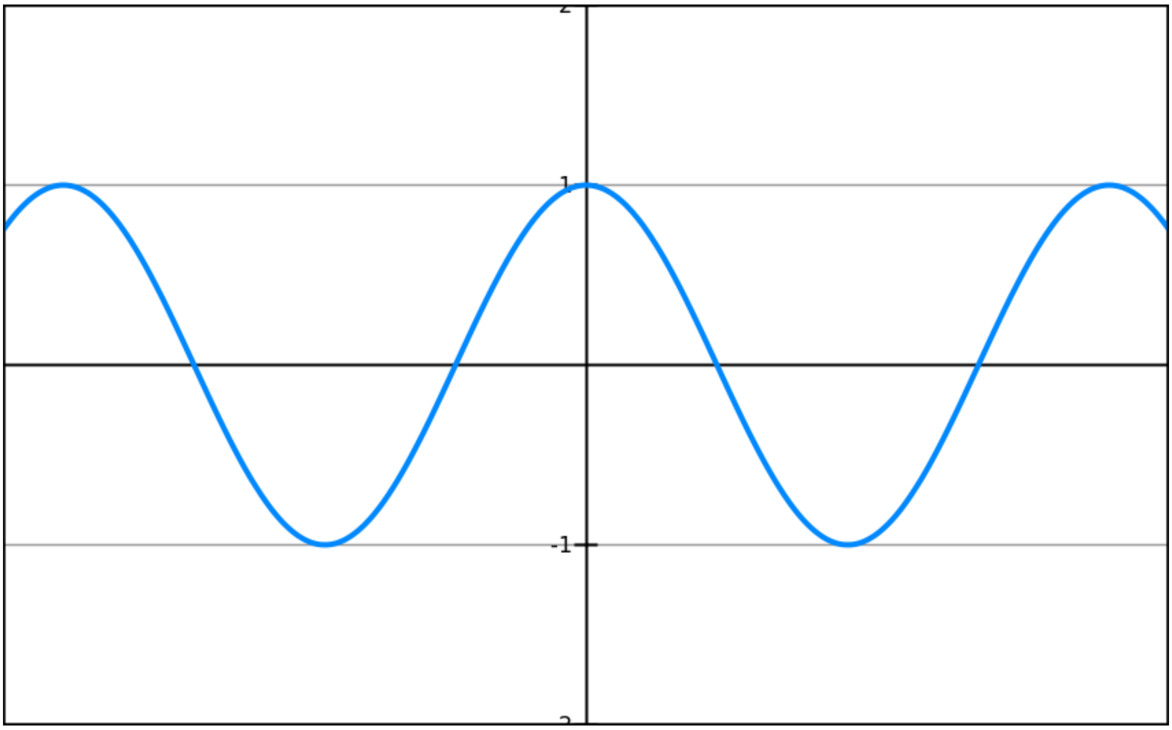


Suppose we know that **cos(60) = 0.5**. By thinking about symmetry in the graph, how could we work out:

Cos(120)

Cos(-60)

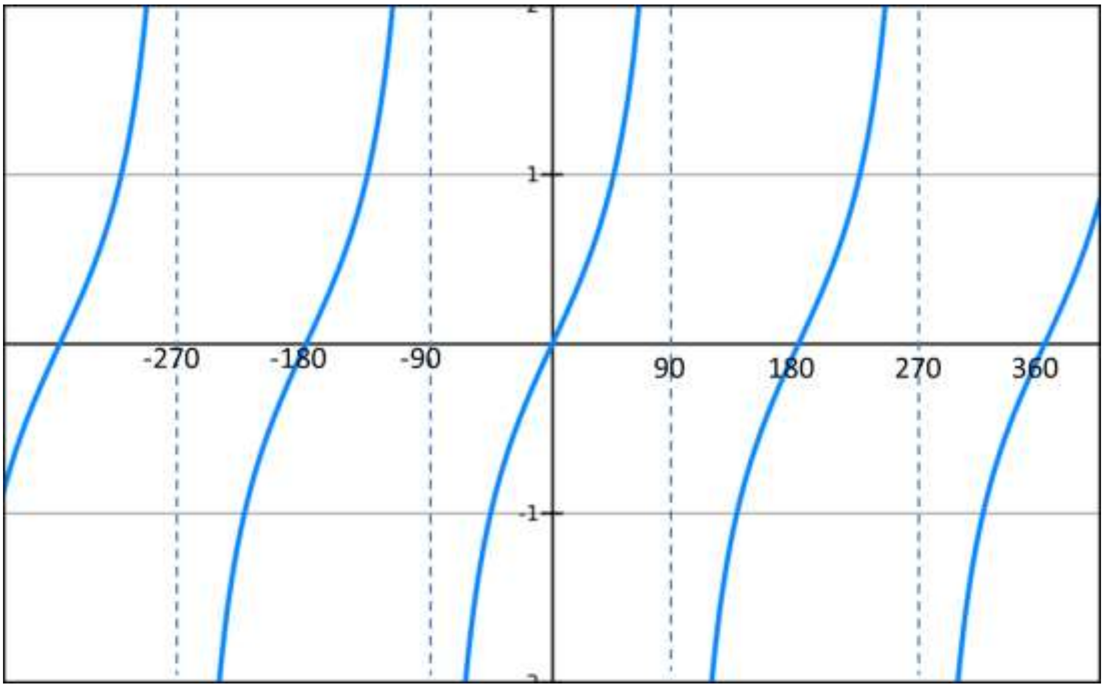
Cos(240)



Suppose we know that $\tan(30°) = \frac{1}{\sqrt{3}}$. By thinking about symmetry in the graph, how could we work out:

Tan(-30)

Tan(150)



Transforming Trig Graphs

We can use our knowledge of transforming graphs to transform trig graphs.

Recap

Examples

1. Sketch $y = 4\sin x$, $0 \leq x \leq 360°$

2. Sketch $y = \cos(x + 45°)$, $0 \leq x \leq 360°$

3. Sketch $y = -\tan x$, $0 \leq x \leq 360°$

4. Sketch $y = \sin\left(\frac{x}{2}\right)$, $0 \leq x \leq 360°$

Extension

1.

[MAT 2013 1B] The graph of $y = \sin x$ is reflected first in the line $x = \pi$ and then in the line $y = 2$. The resulting graph has equation:

A) $y = \cos x$

B) $y = 2 + \sin x$

C) $y = 4 + \sin x$

D) $y = 2 - \cos x$

2.

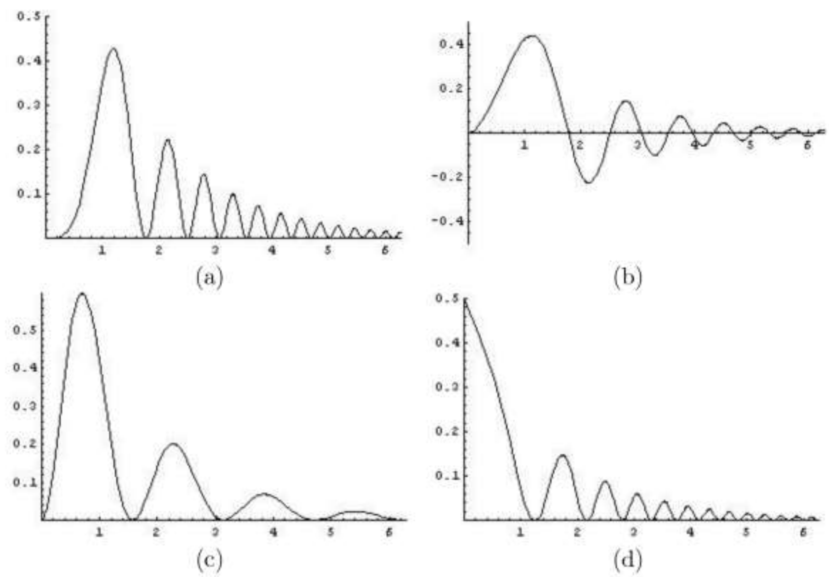
[MAT 2011 1D] What fraction of the interval
$0 \leq x \leq 360°$ is one (or both) of the inequalities:

$\sin x \geq \frac{1}{2}$, $\sin 2x \geq \frac{1}{2}$ true?

3.

MAT 2007 1G] On which of the axes is a sketch of the graph

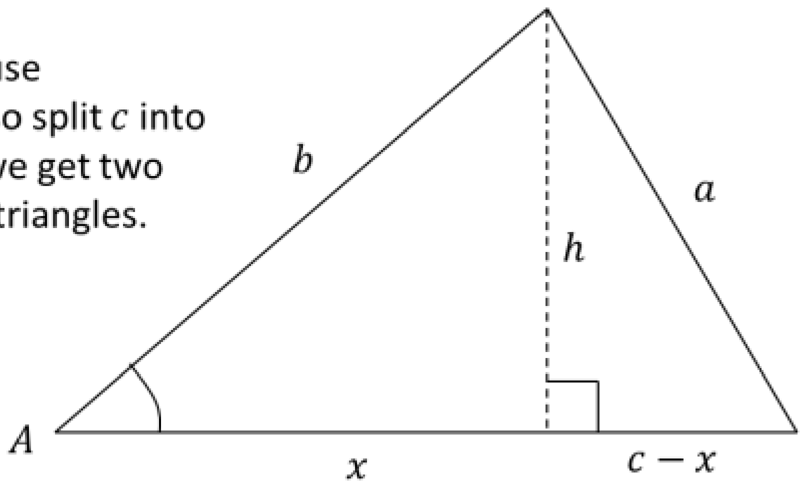
$$y = 2^{-x}\sin^2(x^2)$$



Ex 9F/G Pg 194 – 197.

Proof of Cosine Rule

We want to use Pythagoras, so split c into two so that we get two right-angled triangles.



Proof of Sine Rule

The idea is that we can use the common length of ΔACX and $\angle XBC$, i.e. h, to connect the two triangles, and therefore connect their angles/length.