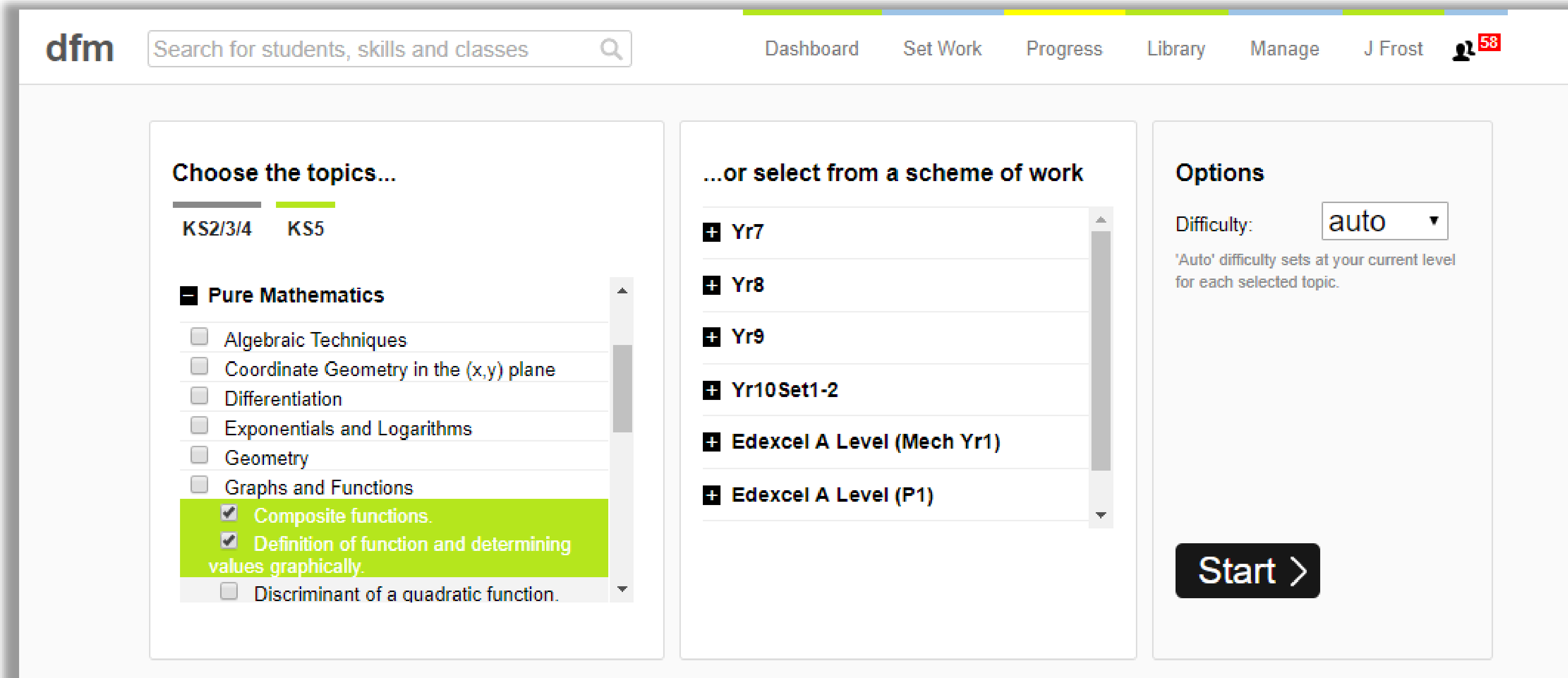
Stats1 Chapter 5 :: Probability

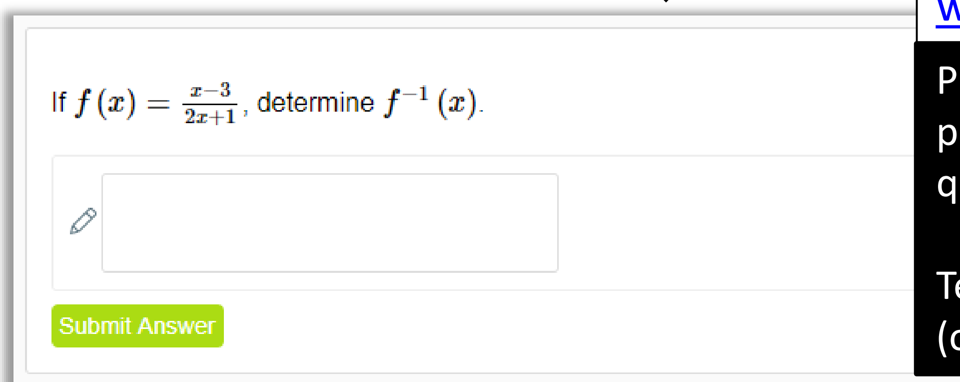
jfrost@tiffin.kingston.sch.uk

www.drfrostmaths.com
@DrFrostMaths

Last modified: 18th January 2019

Use of DrFrostMaths for practice





Register for **free** at:
www.drfrostmaths.com/homework

Practise questions by chapter, including past paper Edexcel questions and extension questions (e.g. MAT).

Teachers: you can create student accounts (or students can register themselves).

Experimental

i.e. Dealing with collected data.

Chp1: Data Collection

Methods of sampling, types of data, and populations vs samples.

Chp2: Measures of Location/Spread

Statistics used to summarise data, including mean, standard deviation, quartiles, percentiles. Use of linear interpolation for estimating medians/quartiles.

Chp3: Representation of Data

Producing and interpreting visual representations of data, including box plots and histograms.

Chp4: Correlation

Measuring how related two variables are, and using linear regression to predict values.

Theoretical

Deal with probabilities and modelling to make inferences about what we 'expect' to see or make predictions, often using this to reason about/contrast with experimentally collected data.

Chp5: Probability

Venn Diagrams, mutually exclusive + independent events, tree diagrams.

Chp6: Statistical Distributions

Common distributions used to easily find probabilities under certain modelling conditions, e.g. binomial distribution.

Chp7: Hypothesis Testing

Determining how likely observed data would have happened 'by chance', and making subsequent deductions.

This Chapter Overview

This chapter is a recap of the concepts you learnt at GCSE.

1 :: Basic Probability

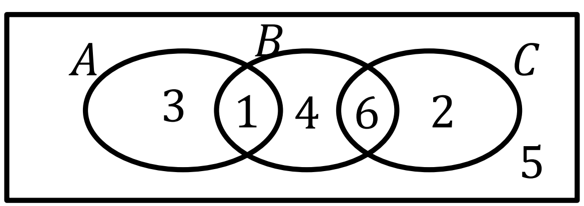
"I throw two fair die. Calculate the probability the sum of the two dice is more than 6."

2 :: Venn Diagrams

"Out of 50 students, 12 play both piano and drums, 30 play piano and 25 play drums. Find the probability a randomly chosen student plays neither instrument."

3 :: Mutually Exclusive/Independent Events

Determine whether A and B are independent.



4 :: Tree Diagrams

"The probability I hit a target is 0.3. If I hit it, the probability I hit again on the next shot is 0.4. If I miss, the probability I hit on the next shot is 0.1. If I shoot 3 times, what's the probability I hit on the first and third shot?"

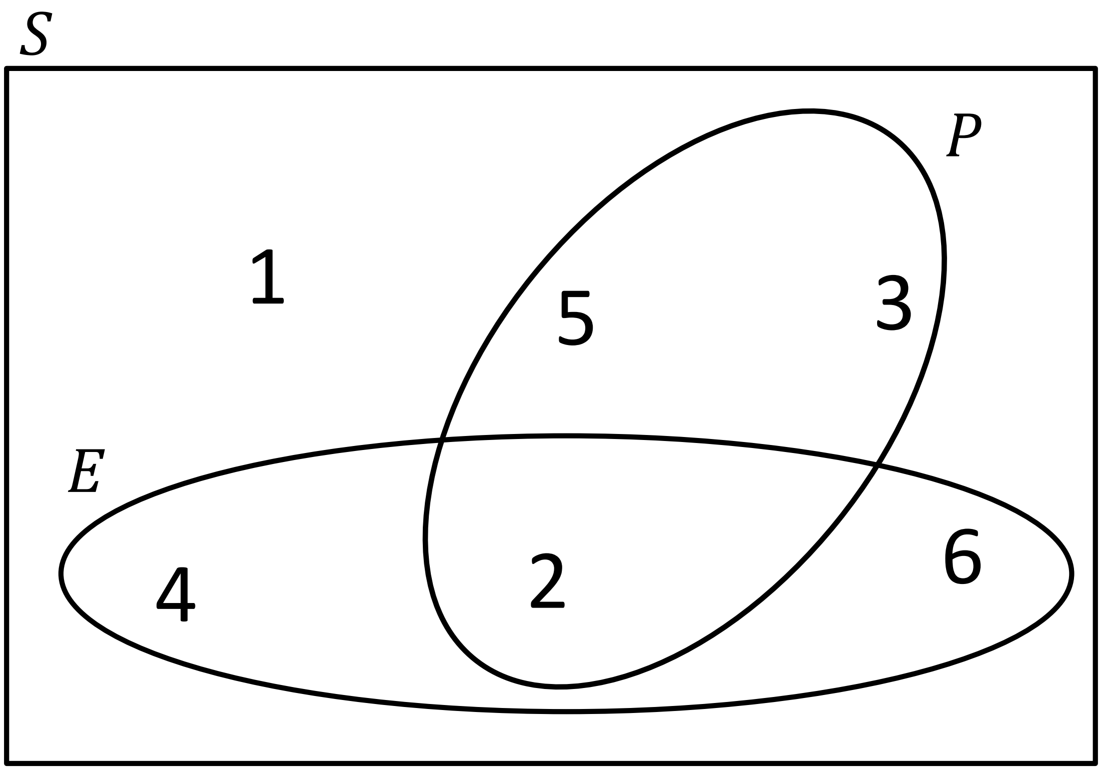
Changes since the old 'S1' syllabus: Conditional probabilities and the Addition Rule have been moved to Year 2. There is also no longer any use of set notation, e.g. ∩ and ∪.

Probability concepts

✎ An **experiment** is a repeatable process that gives rise a number a number of **outcomes**.

✎ An **event** is a set of <u>one or more</u> of these outcomes.

(We often use capital letters to represent them)



$E =$ "rolling an even number"
$P =$ "rolling a prime number"

✎ A **sample space** is the set of all possible outcomes.

Because we are dealing with sets, we can use a **Venn diagram**, where

- the numbers are the individual outcomes,
- the sample space is a rectangle and
- the events are sets, each a subset of the sample space.

You do not need to use set notation like ∩ and ∪ in this module (but ordinarily you would!)

Example

Two fair spinners each have four sectors numbered 1 to 4. The two spinners are spun together and the sum of the numbers indicated on each spinner is recorded.
Find the probability of the spinners indicating a sum of
(a) exactly 5 (b) more than 5

Spinner 1

Spinner 2 / +	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

If the sample space is the amalgamation of two underlying experiments, a table is a helpful way to list the outcomes.

$P(5) =$?

$P(>5) =$?

Another Example

The table shows the times taken, in minutes, for a group of students to complete a number puzzle.

Time, t (min)	$5 \leq t < 7$	$7 \leq t < 9$	$9 \leq t < 11$	$11 \leq t < 13$	$13 \leq t < 15$
Frequency	6	13	12	5	4

A student is chosen at random. Find the probability for a group of students to complete a number puzzle
(a) In under 9 minutes (b) in over 10.5 minutes.

$P(\leq 9) =$?

$P(\geq 10.5) =$?

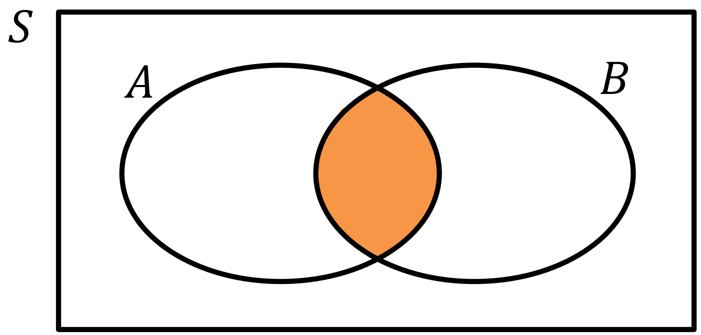
You're effectively doing **linear interpolation** here: 10.50 to 11 is a quarter of the $9 \leq t < 11$ interval, therefore we use a quarter of the 12 people.
This is an estimate because we're assuming the people are equally distributed across the interval.

Exercise 5A

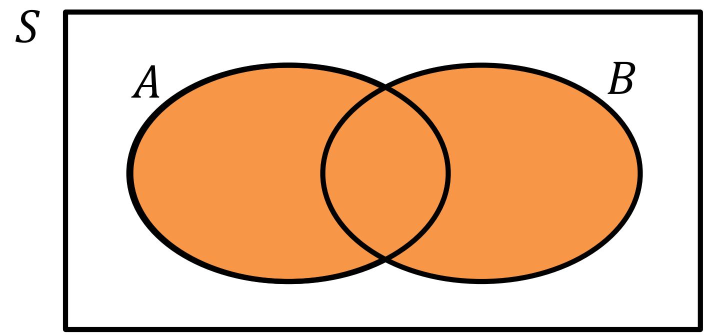
Pearson Pure Mathematics Year 1/AS
Pages 71-72

Venn Diagrams

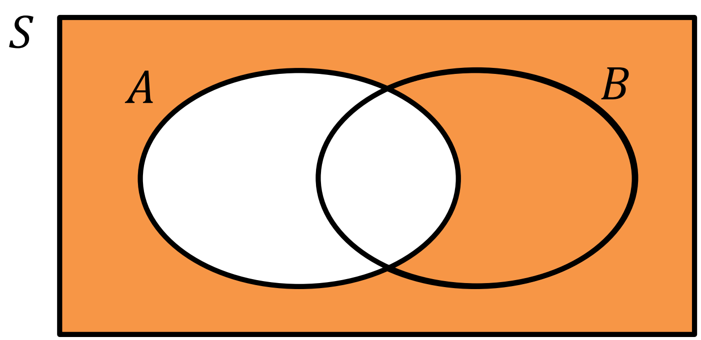
Venn Diagrams allow us to combine events, e.g. "A happened **and** B happened".



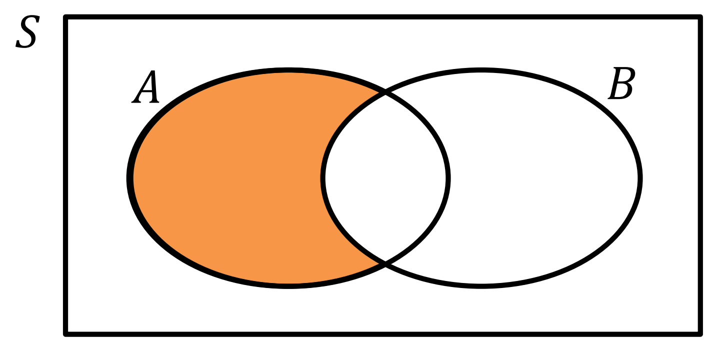
The event "A **and** B"
Known as the **intersection** of A and B.



The event "A **or** B"
Known as the **union** of A and B.



The event "not A"
Known as the **union** of A and B.



These can be combined,
e.g. "A and not B".

Example involving probabilities

We can either put frequencies or probabilities into the Venn Diagram.

Given that $P(A) = 0.6$ and $P(A\ or\ B) = 0.85$, find the probability of:

a) $P(not\ A\ and\ B)$

b) $P(neither\ A\ nor\ B)$

?

Example involving frequencies

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

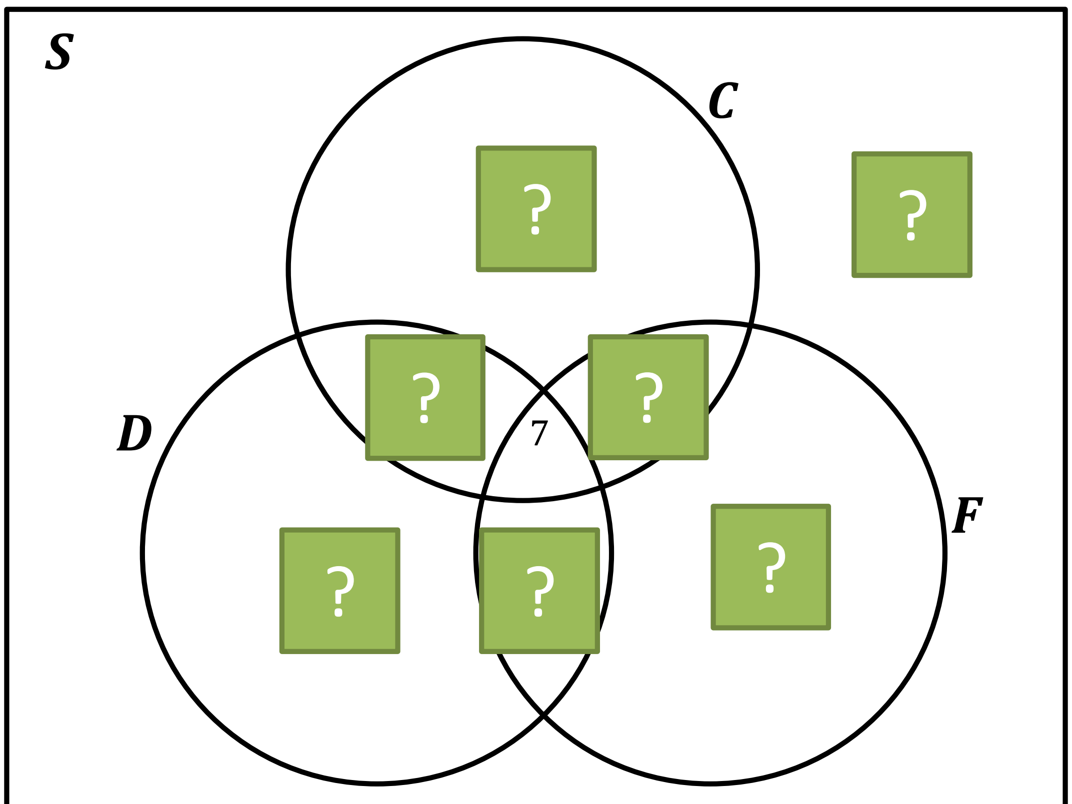
Fill in this Venn Diagram, and hence answer the following questions:

a) $P(owns\ dog\ only)$

b) $P(does\ not\ own\ tropical\ fish)$

c) $P(does\ not\ own\ dogs,\ cats, or\ tropical\ fish)$

Dr Frost's cat "Pippin"



Fro Tip: Start from the centre frequency and work your way outwards using subtraction.

a) ?

b) ?

c) ?

Test Your Understanding

Jan 2012 Q6

The following shows the results of a survey on the types of exercise taken by a group of 100 people.

65 run	48 swim
60 cycle	40 run and swim
30 swim and cycle	35 run and cycle
25 do all three	

(*a*) Draw a Venn Diagram to represent these data. **(4)**

Find the probability that a randomly selected person from the survey

(*b*) takes none of these types of exercise, **(2)**
(*c*) swims but does not run, **(2)**
(d) takes at least two of these types of exercise. **(2)**

~~Jason is one of the above group. Given that Jason runs,~~
~~(*e*) find the probability that he swims but does not cycle.~~ **(3)**

Fro Tip: You'll lose a mark if you don't have a box!

(a) ?

(b) ?

(c) ?

(d) ?

Exercise 5B

Pearson Pure Mathematics Year 1/AS
Pages 74-75

Mutually Exclusive Events

- If two events are mutually exclusive ?
- If A and B are mutually exclusive then:
  - $\boldsymbol{P(A\ and\ B) =}$?
  - $\boldsymbol{P(A\ or\ B) =}$?
- The Venn Diagram would look like:

?

Independent Events

- If two events are independent

?

- If A and B are independent then:
  - $\boldsymbol{P(A\ and\ B)} =$?

Fro Note: Independence does not affect how the circles interact in a Venn Diagram.

Example

1 2 3 4

1 I pick one of the four numbers 1, 2, 3, 4 at random. What's the probability that:

a) I pick a multiple of 2: ?

b) I pick a multiple of 4: ?

2 Explain (conceptually) why these two events are not independent.

?

3 Show that the events are not independent.

?

Further Examples

[Textbook] Events A and B are mutually exclusive and $P(A) = 0.2$ and $P(B) = 0.4$.

a) Find $P(A\ or\ B)$

b) Find $P(A\ but\ not\ B)$

c) Find $P(neither\ A\ nor\ B)$

?

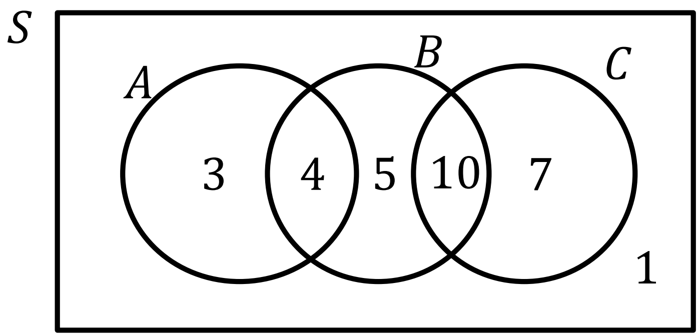
[Textbook] Events A and B are independent and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$. Find $P(A\ and\ B)$.

?

[Textbook] The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes.

a) Find the probability that a student chosen at random watches B or C or both.

b) Determine whether watching A and watching B are statistically independent.



?

Test Your Understanding

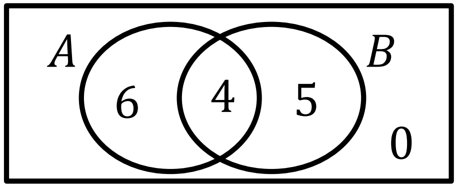
There are three events A, B, C. The events A and B are mutually exclusive.

a) Draw a Venn diagram which represents this information.

b) If $P(A) = 0.1$ and $P(B) = 0.6$, determine $P(neither\ A\ nor\ B)$

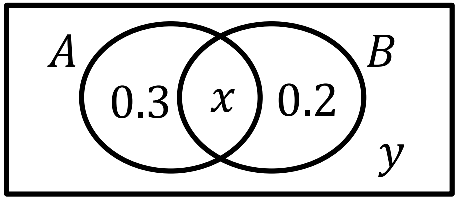
?

The Venn diagram shows the number of people who like each of two different colours. Determine if A and B are independent.



?

The Venn diagram shows the probability of each event. Given that A and B are independent, determine the possible values of x.



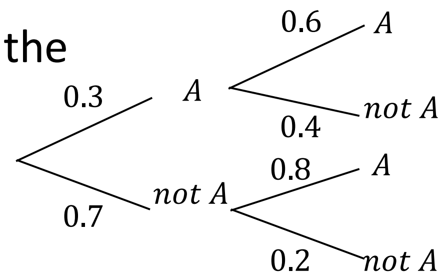
?

Exercise 5C

Pearson Pure Mathematics Year 1/AS
Pages 77-78

Tree Diagrams

At GCSE we saw that tree diagrams were an effective way of showing the outcome of two events which happen **in succession**.
(Personal opinion however is that their use is easily avoidable)



There are 3 yellow and 2 green counters in a bag. I take two counters at random. Determine the probability that:

a) They are of the same colour.
b) They are of different colours.

a

?

I like to list out the matching sequences of outcomes first, then find the probability for each.

b

?

The probability I hit a target on each shot is 0.3. I keep firing until I hit the target. Determine the probability I hit the target on the 5th shot.

?

Exercise 5D

Pearson Pure Mathematics Year 1/AS
Pages 79-80

Extension Questions

1 [STEP I 2010 Q12] Prove that, for any real numbers x and y, $x^2 + y^2 \geq 2xy$.

(i) Carol has two bags of sweets. The first bag contains a red sweets and b blue sweets, whereas the second bag contains b red sweets and a blue sweets. Carol shakes the bags and picks one sweet from each bag without looking. Prove that the probability that the sweets are of the same colour cannot exceed the probability that they are of different colours.

(ii) Simon has three bags of sweets. The first bag contains a red sweet, b white sweets and c yellow sweets. The second bag contains b red sweets, c white sweets and a yellow sweets. The third bag contains c red sweets, a white sweets and b yellow sweets. Simon shakes the bags and picks one sweet from each bag without looking. Show that the probability that exactly two of the sweets are of the same colour is

$$\frac{3(a^2b + b^2c + c^2a + ab^2 + bc^2 + ca^2)}{(a+b+c)^3}$$

and find the probability that the sweets are all of the same colour. Deduce that the probability that exactly two of the sweets are of the same colour is at least 6 times the probability that the sweets are all of the same colour.

2 [STEP I 2011 Q12] I am selling raffle tickets for £1 per ticket. In the queue for tickets, there are m people each with a single £1 coin and n people each with a single £2 coin. Each person in the queue wants to buy a single raffle ticket and each arrangement of people in the queue is equally likely to occur. Initially, I have no coins and a large supply of tickets. I stop selling tickets if I cannot give the required change.

(i) In the case $n = 1$ and , $m \geq 1$, find the probability that I am able to sell one ticket each person in the queue.

(ii) By considering the first people in the queue, show that the probability that I am able to sell one ticket to each person in the queue in the case $n = 2$ and $m \geq 2$ is $\frac{m-1}{m+1}$

(iii) Show that the probability that I am able to sell one ticket to each person in the queue in the case $n = 3$ and $m \geq 3$ is $\frac{m-2}{m+1}$.

3 I have an unfair coin with a fixed probability p of heads. Determine how the unfair coin could be used to simulate a fair coin, i.e. you declare "Heads" or "Tails" each with probability 0.5.

?