MechYr1 Chapter 10 :: Forces and Motion

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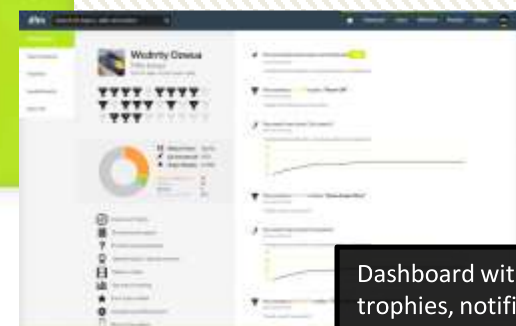

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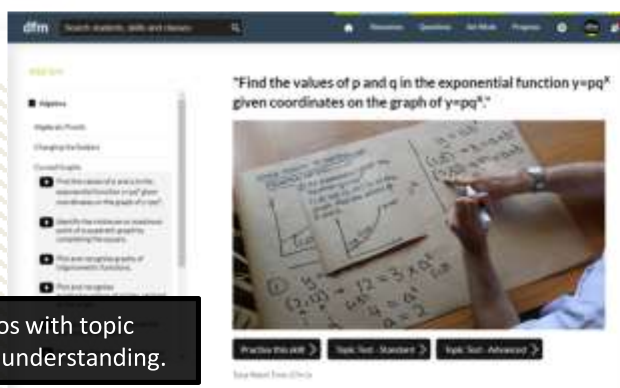

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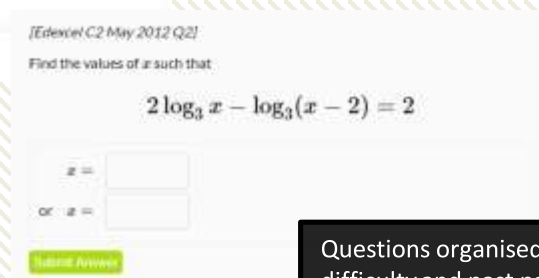
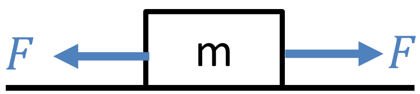


Questions organised by topic, difficulty and past paper.

Newton's Laws of Motion

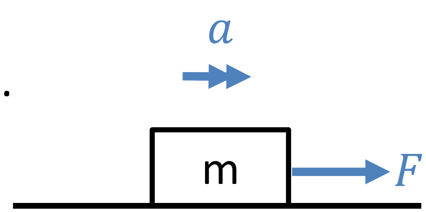
1. An object will remain at rest or continue to move **in a straight line at a constant speed** unless it is acted upon by a **resultant force**
2. The force needed to accelerate a particle is equal to the product of its mass and the acceleration of the particle: **F = ma**
3. Every action has an equal and opposite reaction

1.



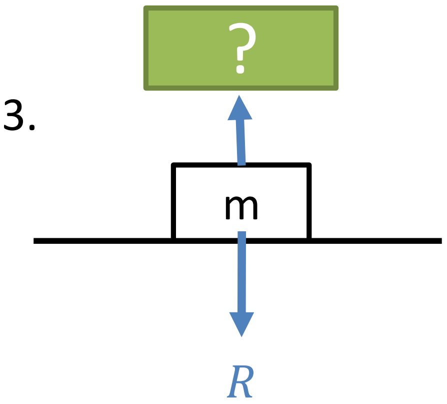
Speed = ? or
Speed = ?

2.

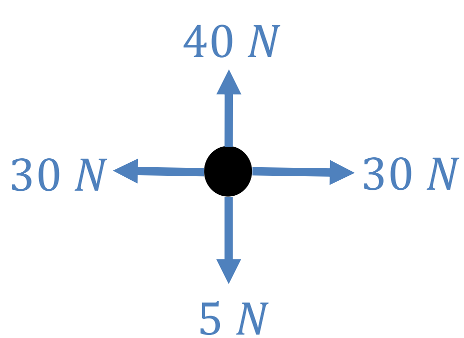


?

3.



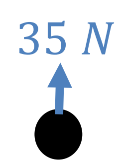
The '**resultant force**' is the overall force acting on the object. An object will accelerate in the direction of the resultant force.



$R(\uparrow)$: ?

$R(\rightarrow)$: ?

We use $R(\ )$ to 'resolve' the forces in a particular direction. **This is standard notation expected in exams.**

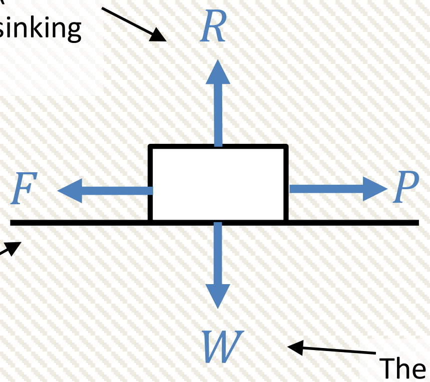


Therefore a 'resultant' force of 35 N upwards and the object will accelerate upwards.

Force Diagrams and Common Forces

We consider the forces acting on each object one at a time.

The **reaction force** of the plane on the block (i.e. resisting the block sinking into the plane)



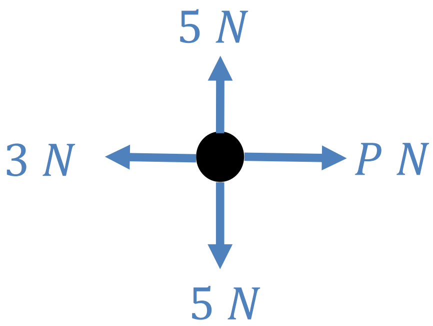
Resistances to motion, in this case the **friction** between the block and the plane.

Force pulling the block. When a string/cable is involved, this is tension T.

The weight of the block.

Recall that we often model an object as a **particle**, i.e. a point with negligible dimensions.

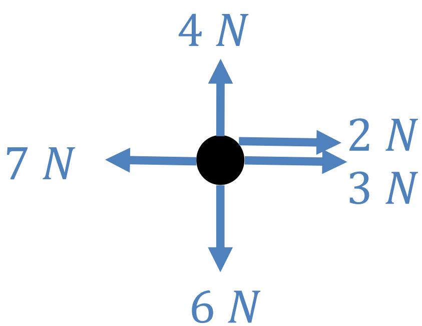
Quickfire Examples



$R(\uparrow)$: ?

$R(\rightarrow)$: ?

? Resultant Force



$R(\uparrow)$: ?

$R(\rightarrow)$: ?

? Resultant Force

Exercise 10A

Pearson Stats/Mechanics Year 1

Pages 158 - 159

Forces as Vectors

Forces have direction, and therefore we can naturally write them as vectors, either in ***i*-*j*** notation or as column vectors.

✎ You can find the resultant of two or more forces given as vectors by adding the vectors.

The forces $(3\boldsymbol{i} - 4\boldsymbol{j})$, $(2\boldsymbol{i} + 5\boldsymbol{j})$ and $(a\boldsymbol{i} + b\boldsymbol{j})$ act on a particle in equilibrium. Find the values of a and b.

?

The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.

(b) Work out the magnitude and bearing of the resultant force.

a ?

b ?

Test Your Understanding

Edexcel M1 Jan 2012 Q3

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ acting on a particle P are given by

$$\mathbf{F}_1 = (7\mathbf{i} - 9\mathbf{j})\text{ N}$$
$$\mathbf{F}_2 = (5\mathbf{i} + 6\mathbf{j})\text{ N}$$
$$\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})\text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q. **(3)**

The force $\mathbf{F}_3$ is now removed. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Find

(b) the magnitude of $\mathbf{R}$, **(2)**

(c) the angle, to the nearest degree, that the direction of $\mathbf{R}$ makes with $\mathbf{j}$. **(3)**

(a) ?

(b) ?

(c) ?

Edexcel M1 May 2009 Q2

A particle is acted upon by two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, given by

$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j})$ N,

$\mathbf{F}_2 = (p\mathbf{i} + 2p\mathbf{j})$ N, where p is a positive constant.

(a) Find the angle between $\mathbf{F}_2$ and $\mathbf{j}$. **(2)**

The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{R}$. Given that $\mathbf{R}$ is parallel to $\mathbf{i}$,

(b) find the value of p. **(4)**

Fro Tip: If a vector is parallel to say $\begin{pmatrix}1\\2\end{pmatrix}$, then it could be any multiple of it, i.e. $\begin{pmatrix}k\\2k\end{pmatrix}$

(a) ?

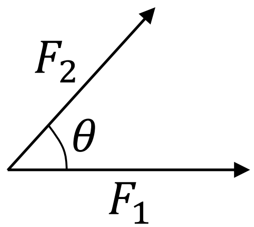
(b) ?

Exercise 10B

Pearson Stats/Mechanics Year 1
Pages 161-162

Extension

A force F_1 acts in the direction of i and a force F_2 acts at an angle of θ to i, as shown. Show that the resultant force has magnitude $\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$



?

Forces and Acceleration

Newton's 2nd Law of Motion: $F = ma$
(where the force F and acceleration a are in the same direction)

This 'feels' right: if we doubled the force, we double the rate at which it accelerates. Similarly, if we have an object of twice the mass, we'd require twice the force to make it accelerate at the same rate.

A car of 2000kg has a driving force of 800N and forces of 200N resisting its motion. Determine its acceleration.

?

A child has a mass of 50kg. What is the gravitational force acting on the child? (i.e. its weight)

?

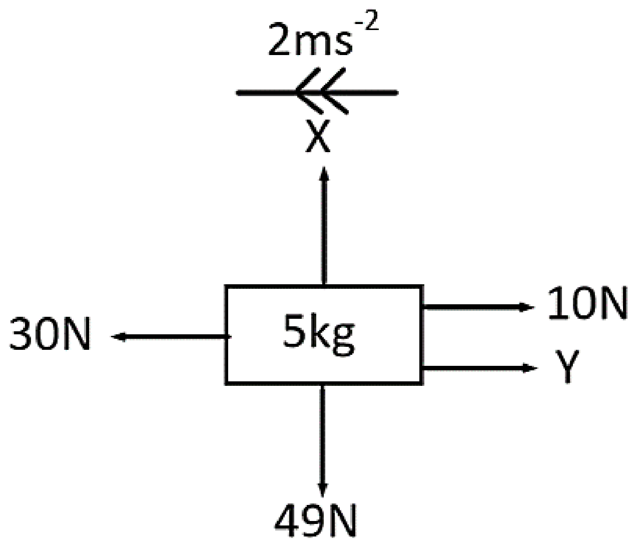
A falling sheep of mass 70kg experiences air resistance of 300 N. Determine the sheep's acceleration as it plummets towards the ground.

?

Combining $F = ma$ with $suvat$ equations

Since $F = ma$ involves both force and acceleration, it allows us to connect calculations involving forces with a calculations involving $suvat$ values.

The forces acting on a body cause it to accelerate as indicated.
a) Find the values of X and Y
b) Find the distance travelled in the first 4 seconds if the object starts at rest.



a

?

No vertical acceleration

$F = ma$

b

?

Forces Acting Under Gravity

Acceleration due to gravity is $g = 9.8$ ms^{-2}

A lift of mass 600kg is raised or lowered by means of a cable attached to its top. When carrying passengers whose total mass is 400kg, the lift accelerates uniformly from rest to 2ms^{-1} over a distance of 5m. Find:

a) The magnitude of the acceleration

b) The tension in the cable if the motion takes place vertically upwards

c) The tension in the cable if the motion takes place vertically downwards

Diagram?

a

?

Show the +ve direction

b

?

$F = ma$

c

?

Testing Your Understanding

Edexcel M1 May 2012 Q5 (abridged)

A particle P is projected vertically upwards from a point A with speed u m s^{-1}. The point A is 17.5 m above horizontal ground. The particle P moves freely under gravity until it reaches the ground with speed 28 m s^{-1}.

The ground is soft and, after P reaches the ground, P sinks vertically downwards into the ground before coming to rest. The mass of P is 4 kg and the ground is assumed to exert a constant resistive force of magnitude 5000 N on P.

(c) Find the vertical distance that P sinks into the ground before coming to rest.

(4)

?

Exercise 10C

Pearson Stats/Mechanics Year 1

Pages 164-166

Motion in 2 dimensions

Force and Acceleration can be represented as both scalars and vectors. Therefore Newton's 2nd law can be used in vector form too.

Can be a vector:	?
Scalar only:	?

This naturally means that $\boldsymbol{F} = m\boldsymbol{a}$ works with vectors too.

Forces $\boldsymbol{F}_1$ $(4\boldsymbol{i} - 7\boldsymbol{j})$, and $\boldsymbol{F}_2$ $(-6\boldsymbol{i} + 2\boldsymbol{j})$ and $\boldsymbol{F}_3$ $(4\boldsymbol{j})$ act on a particle of mass 2kg. Find the acceleration of the particle. Find also the magnitude and the bearing of the acceleration.

?

?

Motion in 2 dimensions

A constant force $\boldsymbol{F}$ N acts on a particle of mass 5kg for 8 seconds. The particle is initially at rest and 8 seconds later it has velocity $(12\boldsymbol{i} - 5\boldsymbol{j})$ ms^{-1}. Find $\boldsymbol{F}$.

?

Test Your Understanding

[Textbook] A boat is modelled as a particle of mass 60 kg being acted on by three forces.

$$F_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix} N, \qquad F_2 = \begin{pmatrix} 10p \\ 20q \end{pmatrix} N, \qquad F_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$ ms^{-2}, find the values of p and q.

?

Exercise 10D

Pearson Stats/Mechanics Year 1

Pages 167-169

Connected Particles

Up to now we've only considered one particle at a time.
When we have multiple connected objects moving in the same direction, **they can be considered either as two separate particles, as a single particle.** What assumptions are made?

?

Example

Edexcel M1 May 2009 Q6

A car of mass 800 kg pulls a trailer of mass 200 kg along a straight horizontal road using a light towbar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 400 N and 200 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N. Find

(a) the acceleration of the car and trailer, **(3)**

(b) the magnitude of the tension in the towbar. **(3)**

The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate. Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N,

(c) find the value of F. **(7)**

? Diagram

a

?

b

?

Key Point 1: The tension in a given piece of string is the same in any part of the string (at a given time). The tension acts away from each particle in the direction of the string.

Key Point 2: Recall that R is used for the 'reaction' force, acting perpendicular to the plane. The reaction forces may be different for P and Q, so we used R_1 and R_2.

Example

Edexcel M1 May 2009 Q6

A car of mass 800 kg pulls a trailer of mass 200 kg along a straight horizontal road using a light towbar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 400 N and 200 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N. Find

(a) the acceleration of the car and trailer,

(3)

(b) the magnitude of the tension in the towbar.

(3)

The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate. Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N,

(c) find the value of F.

(7)

? New Diagram

Remember: Thrust is a **PUSH** force

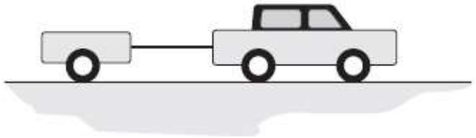
c

?

Connected Particles

Test your understanding

4. A car of mass 900 kg is towing a trailer of mass 100 kg along a horizontal road. There are resistance forces of 60 N and 20 N acting on the car and the trailer respectively.



(a) The engine applies a driving force of 480 N. Calculate
   (i) the acceleration of the car and trailer
   (ii) the tension in the towbar. (4)

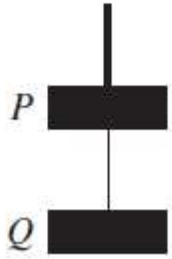
(b) The brakes are now applied, bringing the car to a halt from a speed of 10 ms^{-1} in a distance of 50 m. Find
   (i) the magnitude of the braking force
   (ii) the force in the towbar. (6)

?

Vertical Connected Particles

Example

A brick P of mass 4 kg is suspended by a vertical, light inextensible string. Another brick Q of mass 6 kg is suspended from P by another light inextensible string, as shown in the diagram. The bricks start from rest and are then raised 2 m in 4 seconds. Find the tension in each string.



?

Newton's 3rd Law for Stacked Objects

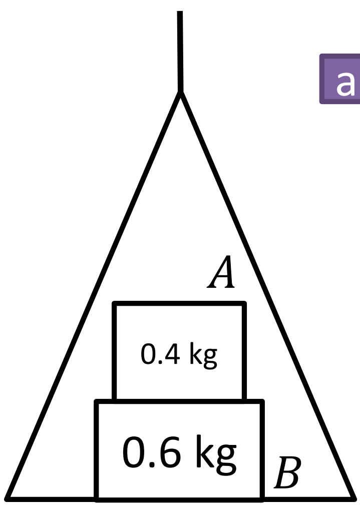
✎ Newton's 3rd Law: For every action there is an equal and opposite reaction.

Therefore when two bodies A and B are in contact, if body A exerts a force on body B, then body B exerts a force on body A that is equal in magnitude and acts in the opposite direction.

Confusion I had as a student: "If the opposite is equal but opposite, surely the object can't move?" Solution: The forces are acting on different objects!

[Textbook] A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400g and the mass of B is 600g. A rests on top of B, as shown in the diagram.
The scale-pan is raised vertically, using the string, with acceleration 0.5 ms^{-2}.

(a) Find the tension in the string.
(b) Find the force exerted on mass B by mass A.
(c) Find the force exerted on mass B by the scale-pan.



a
?

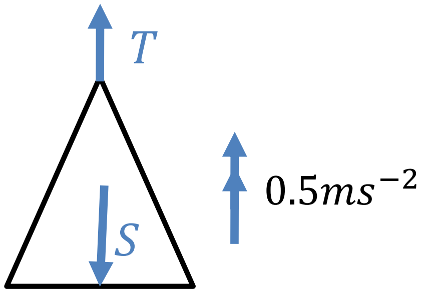
b
?

Newton's 3rd Law for Stacked Objects

[Textbook] A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400g and the mass of B is 600g. A rests on top of B, as shown in the diagram.
The scale-pan is raised vertically, using the string, with acceleration 0.5 ms^{-2}.

(a) Find the tension in the string.
(b) Find the force exerted on mass B by mass A.
(c) Find the force exerted on mass B by the scale-pan.

Fro Tip: When you have objects stacked on top of each other, you would typically resolve forces on the top object, and the two objects combined. It's too complicated to consider forces on the bottom object in isolation. We can use Newton's 3rd Law to reverse "force of A on B" to "force of B on A" and vice versa.



c

?

Test Your Understanding

Edexcel M1 May 2013 Q2

A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 m s^{-2}. By modelling the cable as being light and inextensible, find

(a) the tension in the cable, **(3)**

(b) the magnitude of the force exerted on the woman by the floor of the lift. **(3)**

(a)	?
(b)	?

Exercise 10E

Pearson Stats/Mechanics Year 1
Pages 171-172

Pulleys

A pulley is a wheel on which a rope/string/cable passes.
For the purposes of Mechanics Year 1, the two particles hanging either side will either be horizontal or vertical.

Particles of mass 4kg and 2kg are connected by a light string passing over a smooth, fixed pulley. The particles hang freely and are released from rest.
a) Find the acceleration of the two particles and the tension in the string. Let the acceleration be *a* and the tension in the string be T
b) Find the force exerted on the pulley by the string

Why can't we just model both particles as a single particle as before?

?

Under what conditions is the tension in each part of the string the same?

?

Remember that tension acts away from each object in the direction of the string. This includes the pulley!

Ensure that you add the direction of acceleration at each moving particle, and resolve forces in this direction.

?
Diagram

a

?

b

?

Horizontal and vertical string

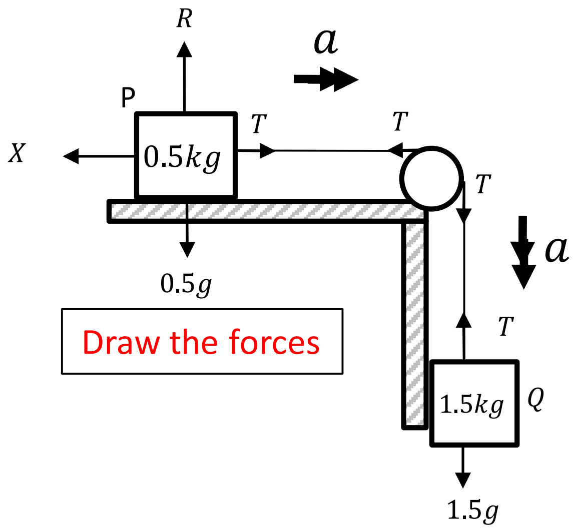
(Take $g = 10ms^{-2}$ in this question)

The diagram shows a particle, P, of mass 0.5kg on a smooth horizontal table. P is connected to another particle, Q, of mass 1.5kg, by a taut, light, inextensible string which passes over a small, fixed, smooth pulley at the edge of the table, Q hanging vertically below the pulley.

A horizontal force of magnitude XN acts on P as shown.

a) Given the system is in equilibrium, find X

b) Given that $X = 12$, find the distance travelled by Q in the first two seconds of its motion, following the release of the system from rest. You may assume that P does not reach the pulley in this time.



Draw the forces

a

?

b

?

Test Your Understanding

Edexcel M1 Jan 2010 Q6

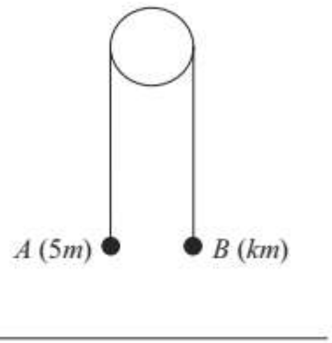


Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.

(3)

(b) Find the value of k.

(3)

(c) State how you have used the information that the pulley is smooth.

(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane.

(7)

("N2L" = Newton's 2nd Law)

(a) ?

(b) ?

(c) ?

(d ?

Exercise 10F

Pearson Stats/Mechanics Year 1

Pages 175-177