P1 Chapter 1 :: Algebraic Expressions

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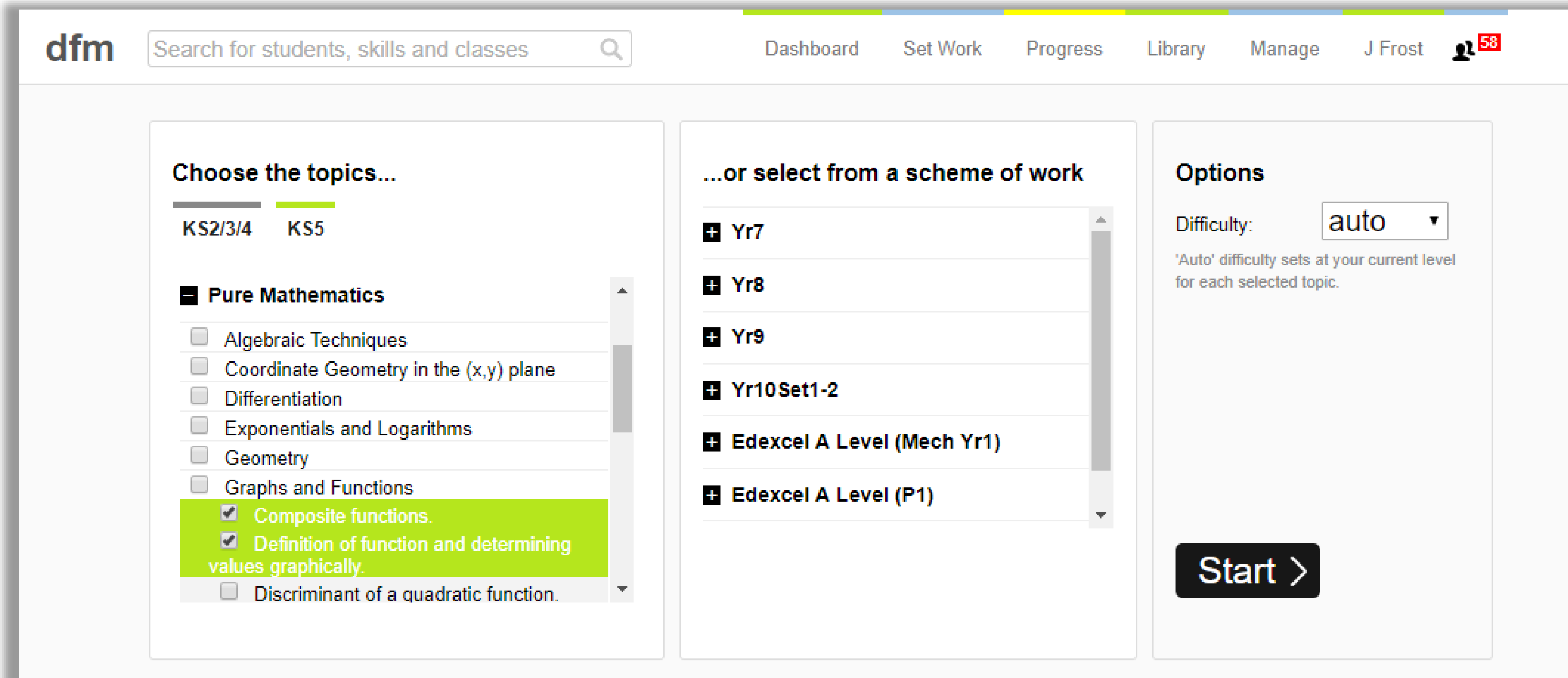

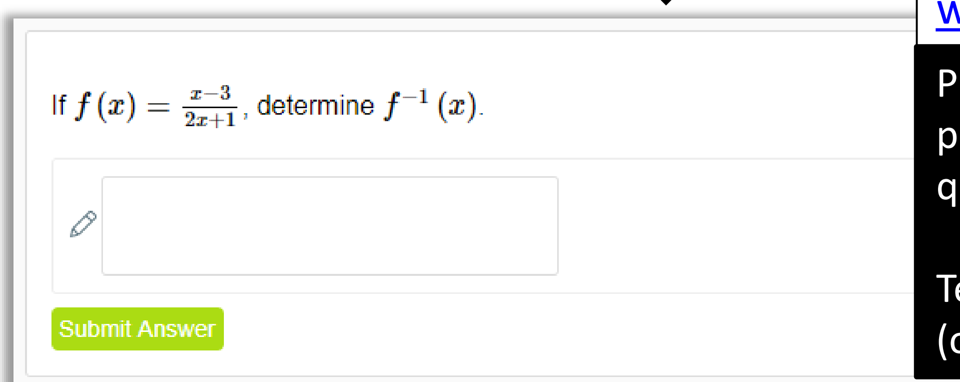

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Practise questions by chapter, including past paper Edexcel questions and extension questions (e.g. MAT).

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Chapter Overview

As a relatively gentle introduction to Pure, most of this chapter is a recap of core GCSE algebraic skills.

1:: Basic Index Laws

Simplify

$$\frac{3x^2 - 6x^5}{2x}$$

2:: Fractional/Negative Powers (change to textbook order)

$$\left(\frac{27}{8}\right)^{-\frac{2}{3}}$$

3:: Factorise quadratics/cubics

Factorise fully:

$$x^3 - 16x$$

NEW! (since GCSE)
You may have to combine factorisation techniques to factorise cubics.

4:: Expand brackets

Expand and simplify

$$(x-3)^2(x+1)$$

5:: Surds

Rationalise the denominator of

$$\frac{1}{3+\sqrt{2}}$$

NEW! (since GCSE)
You've dealt with expressions of the form $\frac{a}{\sqrt{b}}$, but not with more complex denominators such as $\frac{a}{b-\sqrt{c}}$

1 :: Basic Index Laws

We use laws of indices to simplify powers of the same base.

base

$$3^5$$

exponent or power or index (plural: indices)

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n$$

4 Examples:

1. Simplify $(a^3)^2 \times 2a^2$

?

2. Simplify $(4x^3y)^3$

?

3. Simplify $2x^2(3+5x) - x(4-x^2)$

?

Tip: A common student error is to get the sign wrong of $+x^3$

1 :: Basic Index Laws

We use laws of indices to simplify powers of the same base.

base $\quad 3^5 \quad$ exponent or power or index (plural: indices)

$$a^m \times a^n = a^{m+n}$$
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$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n$$

Examples:

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4. Simplify $\frac{x^3-2x}{3x^2}$

There are 2 methods for simplifying fractional expressions.

Method 1 : Split the fraction (preferred by textbook!)

?

Method 2 : Factorise and simplify

?

Test Your Understanding

1 Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

?

2 Simplify $\frac{2x+x^5}{4x^3}$

?

3 Expand and simplify

$$2x(3-x^2)-4x^3(3-x)$$

?

4 Simplify $2^x \times 3^x$

?

Exercise 1A

Pearson Pure Mathematics Year 1/AS
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Extension (Full Database: http://www.drfrostmaths.com/resources/resource.php?rid=268)

1 *[MAT 2006 1A]*

Which of the following numbers is largest?

- $\left(\left(2^3\right)^2\right)^3$
- $\left(2^3\right)^{\left(2^3\right)}$
- $2^{\left(\left(3^2\right)^3\right)}$
- $2^{\left(3^{\left(2^3\right)}\right)}$

?

2 *[MAT 2012 1B]*

Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers.

Then N will definitely be a square number whenever:

- k is even;
- $k+n$ is odd;
- k is odd but $m+n$ is even;
- $k+n$ is even.

?

2 :: Negative and Fractional Indices (later on in textbook)

$$a^0 = 1$$
$$a^{\frac{1}{m}} = \sqrt[m]{a}$$
$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$
$$a^{-m} = \frac{1}{a^m}$$

Note: $\sqrt{9}$ only means the <u>positive</u> square root of 9, i.e. 3 not -3.
Otherwise, what would be the point of the $\pm$ in the quadratic formula before the $\sqrt{b^2-4ac}$?

Prove that $x^{\frac{1}{2}} = \sqrt{x}$

?

Evaluate $27^{-\frac{1}{3}}$

?

Evaluate $32^{\frac{2}{5}}$

?

Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

?

Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

?

If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form ka^n where k, n are constants.

?

Exercise 1D

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Extension (Full Database: http://www.drfrostmaths.com/resources/resource.php?rid=268)

[MAT 2007 1A]

Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$

?

2 :: Expanding Brackets

If you have ever been taught 'FOIL' to multiply brackets please purge it from your mind now – instead:

To expand brackets multiply each term in the first bracket by each term in the second.

$$(x-y)(x+y-1)$$

$$= x^2 + xy - x - xy - y^2 + y$$

$$= x^2 - y^2 - x + y$$

Tip: My order is "first term in first brackets times each in second, then second term in first bracket times each in second, etc."

For more than 2 brackets, multiply two out each time to reduce the number of brackets by one.

Example: $(x+1)(x+2)(x+3)$

?

Test Your Understanding

1

Expand and simplify

$$(x+5)(x-2)(x+1)$$

?

2

Expand and simplify:

$$2(x-3)(x-4)$$

?

3

Expand and simplify:

$$(2x-1)^3$$

?

Exercise 1B

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Extension (Full Database: http://www.drfrostmaths.com/resources/resource.php?rid=268)

1 *[MAT 2002 1B]*

Of the following three alleged algebraic identities, at least one is wrong.

(i) $yz(z-y)+zx(x-z)+xy(y-x)$
$= (z-y)(x-z)(y-x)$

(ii) $yz(z-y)+zx(x-z)+xy(y-x)$
$= (z-y)(z-x)(y-x)$

(iii) $yz(x+y)+zx(z+x)+xy(y+x)$
$= (z+y)(z+x)(y+x)$

Which of the following statements are correct? Tick all that apply.

- (i)
- (ii)
- (iii)

?

2 *[MAT 2007 1E]*

If x and n are integers then

$$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

is:

- negative when $n > 5$ and $x < 5$
- negative when n is odd and $x > 5$
- negative when n is a multiple of 3 and $x > 5$
- negative when n is even and $x < 5$

?

3 :: Factorising

A factorised expression is one which is expressed as **a product of expressions**.

$$x(x+1)(x+2)$$

Factorised as it is the product of 3 linear factors, $x, x+1$ and $x+2$.

Note: A linear expression is of the form $ax+b$. It is called linear because plotting $y=ax+b$ would form a straight line.

$$x(x+1)+(x-1)(x+1)$$

Not factorised because the outer-most operation is a sum, not a product.

Basic Examples:

$x^3+x^2=$?

$4x-8xy=$?

Factorising Quadratics

Recap:

We find two numbers which multiply to give the coefficient of x and multiply to give the constant term.

$$x^2 \overset{\oplus}{-} 5x \overset{\otimes}{-} 14 = \boxed{?}$$

Fro Note: The *coefficient* of a term is the constant on front of it, e.g. the coefficient of $4x^2$ is 4.

But what if the coefficient of x^2 is not 1?

$$2x^2 + 5x - 12 = \boxed{?}$$

The easiest way is to use your common sense to guess the brackets. What multiplies to give the $2x^2$? What multiplies to give the constant term of -12?

Or you can 'split the middle term' (don't be embarrassed if you've forgotten how to!)

$$2x^2 + 5x - 12 \quad \oplus\ \boxed{?} \quad \otimes\ \boxed{?}$$

$$= \boxed{?}$$

$$= \boxed{?}$$

$$= \boxed{?}$$

STEP 1: Find two numbers which add to give the middle number and multiply to give the first times last.

STEP 2: Split the middle term.

STEP 3: Factorise first half and second half ensuring bracket is duplicated..

STEP 4: Factorise out bracket.

Other Factorisations

Difference of two squares:

$$4x^2 - 9 =$$?

Using multiple factorisations:

$$x^3 - x$$

$=$

$=$?

Tip: Always look for a common factor first before using other factorisation techniques.

$$x^3 + 3x^2 + 2x$$

$=$

$=$?

Test Your Understanding

1

Factorise completely:
$$6x^2 + x - 2$$

?

2

Factorise completely:
$$x^3 - 7x^2 + 12x$$

?

☠

Factorise completely:
$$x^4 - 1$$

?

☠

Factorise completely:
$$x^3 - 1$$

?

Exercise 1C

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5 :: Surds

Recap:

A surd is a root of a number that does not simplify to a rational number.

Note: A *rational* number is any which can be expressed as $\frac{a}{b}$ where a, b are integers. $\frac{2}{3}$ and $\frac{4}{1} = 4$ are rational numbers, but π and $\sqrt{2}$ are not.

Laws:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$\sqrt{3} \times 2 =$?

$3\sqrt{5} \times 2\sqrt{5} =$?

$\sqrt{8} = \sqrt{4}\sqrt{2} =$?

$\sqrt{12} + \sqrt{27} =$?

$(\sqrt{8} + 1)(\sqrt{2} - 3) =$?

$=$?

$=$?

Exercise 1E

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Extension (questions used with permission by the UKMT)

1 [SMC 2014 Q24] Which of the following is smallest?

- $10 - 3\sqrt{11}$
- $8 - 3\sqrt{7}$
- $5 - 2\sqrt{6}$
- $9 - 4\sqrt{5}$
- $7 - 4\sqrt{3}$

?

2 [SMC 2012 Q21] Which of the following numbers does *not* have a square root in the form $x + y\sqrt{2}$, where x and y are positive integers?

- $17 + 12\sqrt{2}$
- $22 + 12\sqrt{2}$
- $38 + 12\sqrt{2}$
- $54 + 12\sqrt{2}$
- $73 + 12\sqrt{2}$

?

6 :: Rationalising The Denominator

Here's a surd. What could we multiply it by such that it's no longer an irrational number?

$$\sqrt{5} \times \boxed{?} = \boxed{?}$$

$$\frac{1}{\sqrt{2}} \times \boxed{?} = \boxed{?}$$

In this fraction, the denominator is irrational. '**Rationalising the denominator**' means making the denominator a rational number.

What could we multiply this fraction by to both rationalise the denominator, but leave the value of the fraction unchanged?

Note: There's two reasons why we might want to do this:

1. For aesthetic reasons, it makes more sense to say "half of root 2" rather than "one root two-th of 1". It's nice to divide by something whole!
2. It makes it easier for us to add expressions involving surds.

Examples

$$\frac{3}{\sqrt{2}} = \boxed{?}$$

$$\frac{6}{\sqrt{3}} = \boxed{?}$$

$$\frac{7}{\sqrt{7}} = \boxed{?}$$

$$\frac{15}{\sqrt{5}} + \sqrt{5} = \boxed{?}$$

Test Your Understanding:

$$\frac{12}{\sqrt{3}} = \boxed{?}$$

$$\frac{2}{\sqrt{6}} = \boxed{?}$$

$$\frac{4\sqrt{2}}{\sqrt{8}} = \boxed{?}$$

More Complex Denominators

You've seen 'rationalising a denominator', the idea being that we don't like to divide things by an irrational number.

But what do we multiply the top and bottom by if we have a more complicated denominator?

$$\frac{1}{\sqrt{2}+1} \times \boxed{?} = \boxed{?}$$

We basically use the same expression but with the sign reversed (this is known as the *conjugate*). That way, we obtain the difference of two squares. Since $(a+b)(a-b) = a^2 - b^2$, any surds will be squared and thus we'll end up with no surds in the denominator.

More Examples

$$\frac{3}{\sqrt{6}-2} \times \boxed{?} = \boxed{?}$$

You can explicitly expand out $(\sqrt{6}-2)(\sqrt{6}+2)$ in the denominator, but remember that $(a-b)(a+b) = a^2 - b^2$ so we can mentally obtain $6 - 4 = 2$
Just remember: 'difference of two squares'!

$$\frac{4}{\sqrt{3}+1} \times \boxed{?} = \boxed{?} = \boxed{?}$$

$$\frac{3\sqrt{2}+4}{5\sqrt{2}-7} \times \boxed{?} = \boxed{?}$$

Test Your Understanding

Rationalise the denominator and simplify

$$\frac{4}{\sqrt{5}-2}$$

?

Rationalise the denominator and simplify

$$\frac{2\sqrt{3}-1}{3\sqrt{3}+1}$$

?

AQA IGCSE FM June 2013 Paper 1

Solve $y\left(\sqrt{3}-1\right)=8$

Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

?

Exercise 1F (Page 15)

or alternatively: (not in textbook)

1 Rationalise the denominator and simplify the following:

a $\dfrac{1}{\sqrt{5}+2} =$?

b $\dfrac{\sqrt{3}}{\sqrt{3}-1} =$?

c $\dfrac{\sqrt{5}+1}{\sqrt{5}-2} =$?

d $\dfrac{2\sqrt{3}-1}{3\sqrt{3}+4} =$?

e $\dfrac{5\sqrt{5}-2}{2\sqrt{5}-3} =$?

2 Expand and simplify:

$$(\sqrt{5}+3)(\sqrt{5}-2)(\sqrt{5}+1) = \text{?}$$

3 Rationalise the denominator, giving your answer in the form $a + b\sqrt{3}$.

$$\frac{3\sqrt{3}+7}{3\sqrt{3}-5} = \text{?}$$

4 Solve $x(4-\sqrt{6}) = 10$ giving your answer in the form $a + b\sqrt{6}$.

?

5 Solve $y(1+\sqrt{2}) - \sqrt{2} = 3$

$$y = \frac{3+\sqrt{2}}{1+\sqrt{2}} = \text{?}$$

6 Simplify:

$$\frac{\sqrt{a+1}-\sqrt{a}}{\sqrt{a+1}+\sqrt{a}} = \text{?}$$

A final super hard puzzle

☠ Solve $\frac{\sqrt[4]{9}}{\sqrt[5]{27}} = \sqrt[x]{3}$

?