Stats Yr2 Chapter 3 :: Normal Distribution

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Last modified: 11th February 2018

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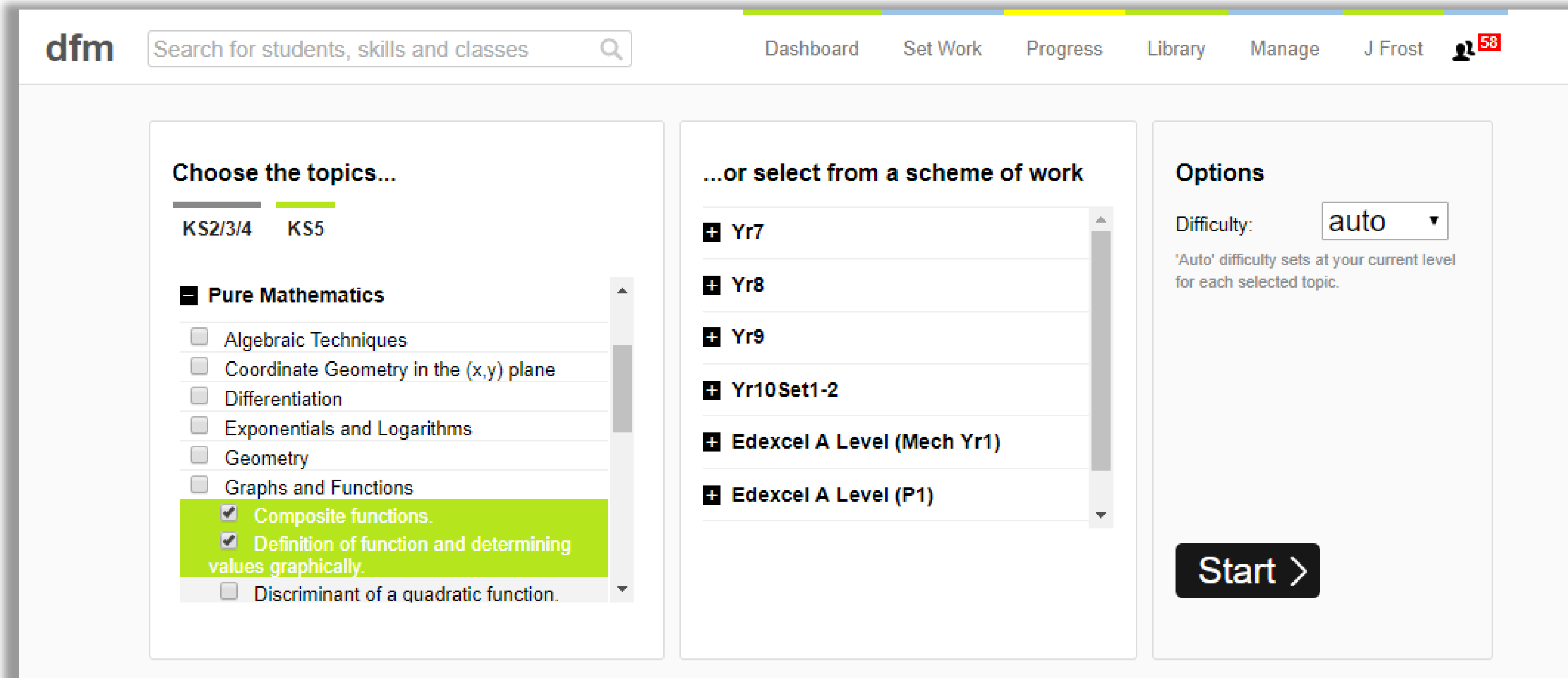

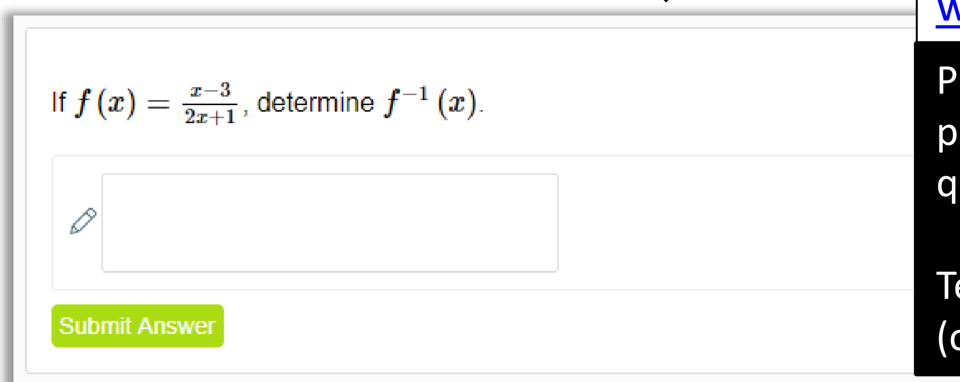

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Chapter Overview

1:: Characteristics of the Normal Distribution

What shape is it? What parameters does it have?

2:: Finding probabilities on a standard normal curve.

"Given that IQ is distributed as $X \sim N(100, 15^2)$, determine the probability that a randomly chosen person has an IQ above 130."

3:: Finding unknown means/standard deviations.

In Wales, 30% of people have a height above 1.6m. Given the mean height is 1.4m and heights are normally distributed, determine the standard deviation of heights.

4:: Binomial → Normal Approximations

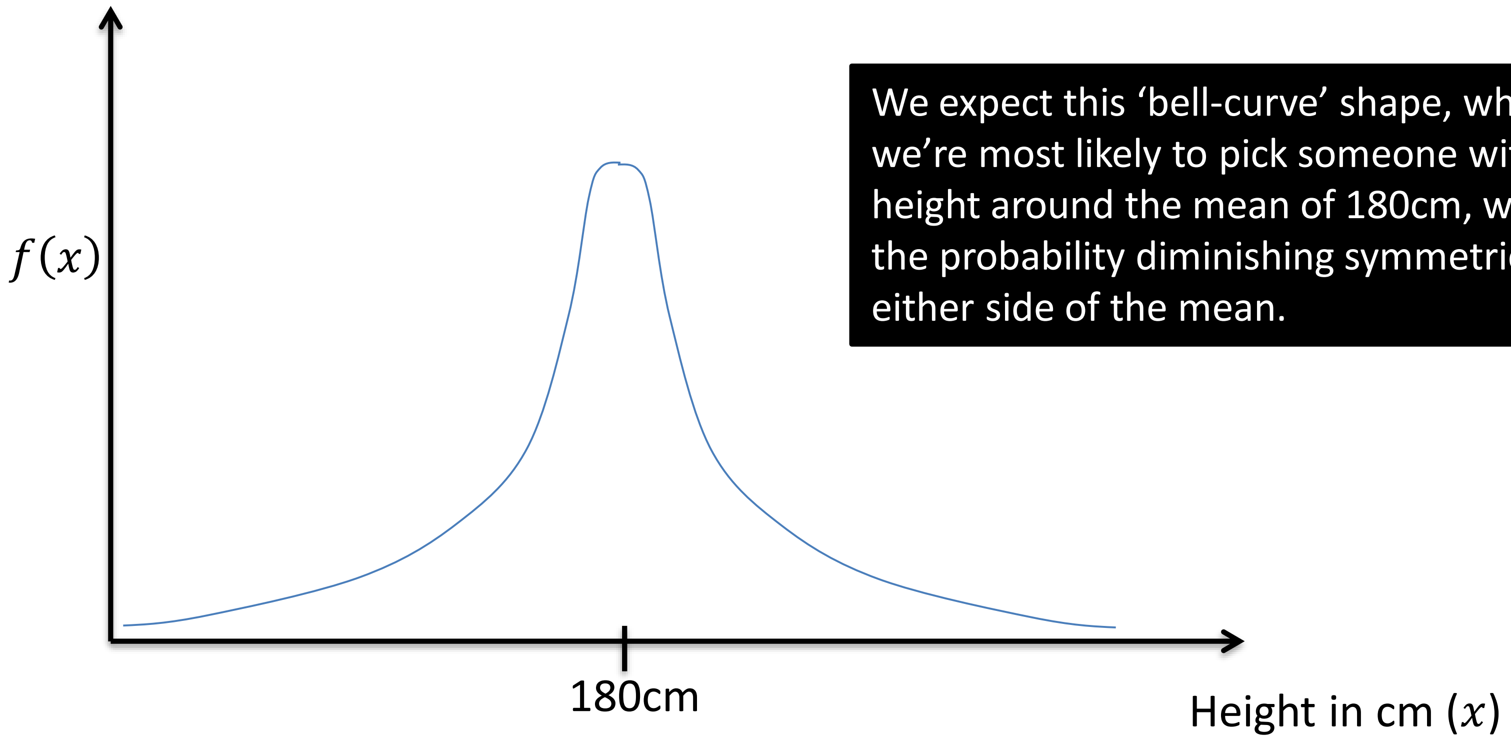
How would I approximate $X \sim B(10, 0.4)$ using a Normal distribution? Under what conditions can we make such an approximation?

5:: Hypothesis Testing

Teacher Notes: This is a combination of all the old S1 content combined with aspects of S2 (Normal approximations) and S3! (hypothesis testing on the mean of a normal distribution)

What does it look like?

The following shows what the probability distribution might look like for a random variable X, if X is the height of a randomly chosen person.

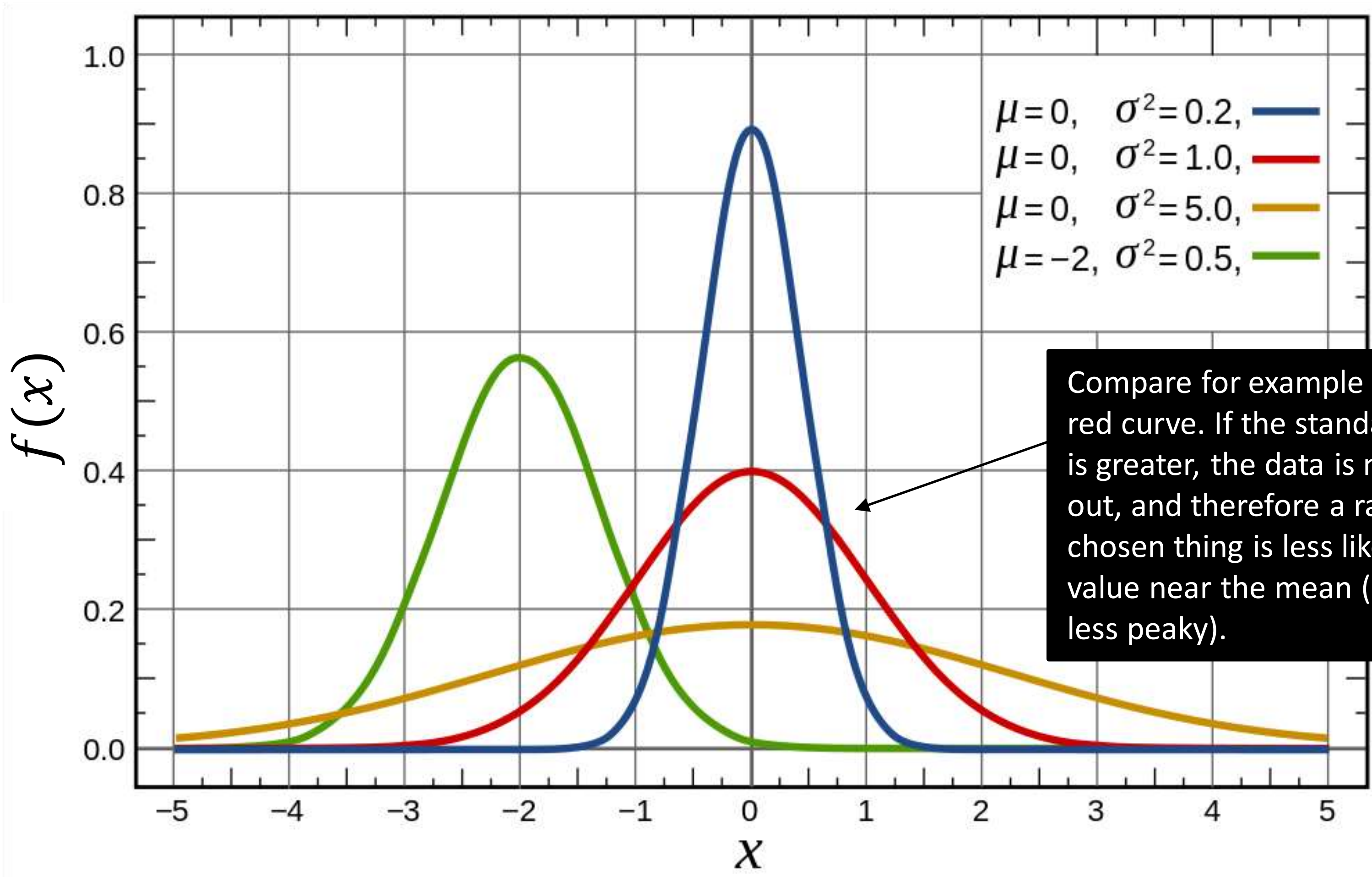


We expect this 'bell-curve' shape, where we're most likely to pick someone with a height around the mean of 180cm, with the probability diminishing symmetrically either side of the mean.

A variable with this kind of distribution is said to have a **normal distribution**.

For normal distributions we tend to draw the y axis at the mean for symmetry.

What does it look like?

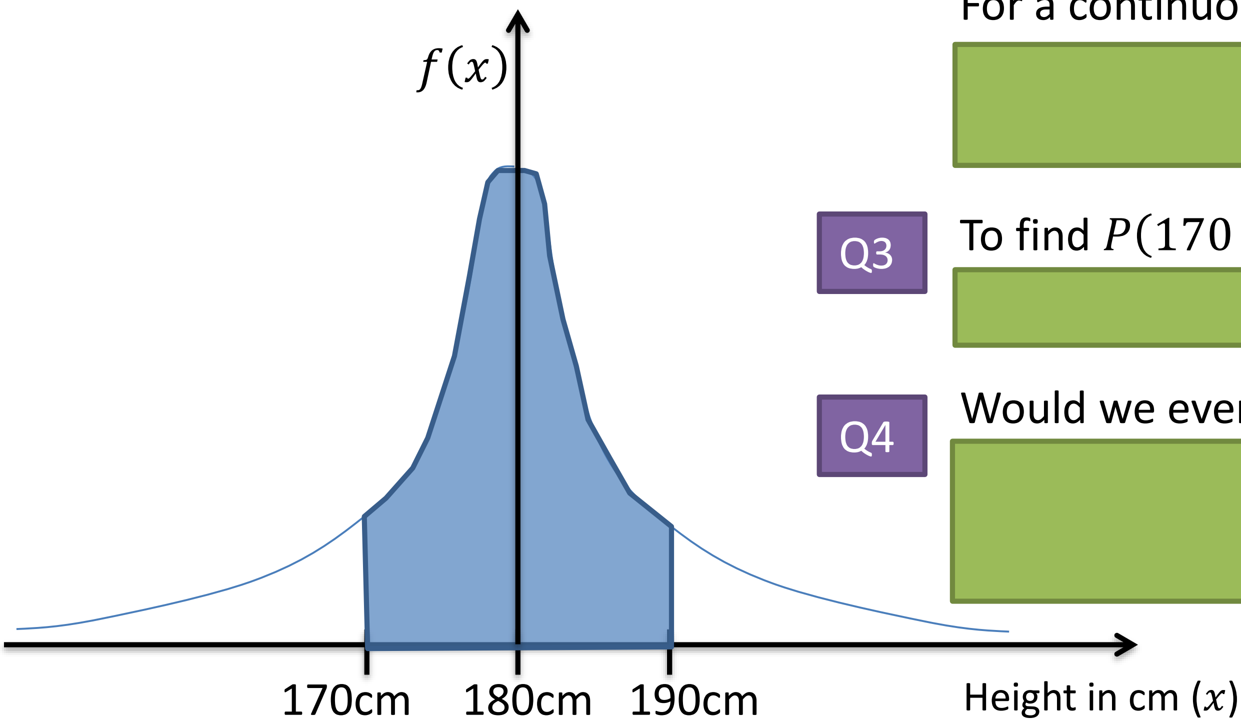


Compare for example the blue and red curve. If the standard deviation is greater, the data is more spread out, and therefore a randomly chosen thing is less likely to have a value near the mean (hence it being less peaky).

We can set the mean μ and the standard deviation σ of the Normal Distribution. If a random variable X is normally distributed, then we write

$$\boldsymbol{X \sim N(\mu, \sigma^2)}$$

Normal Distribution Q & A



Q1 For a Normal Distribution to be used, the variable has to be:

?

Q2 With a discrete variable, all the probabilities had to add up to 1.
For a continuous variable, similarly:

?

Q3 To find $P(170 < X < 190)$, we could:

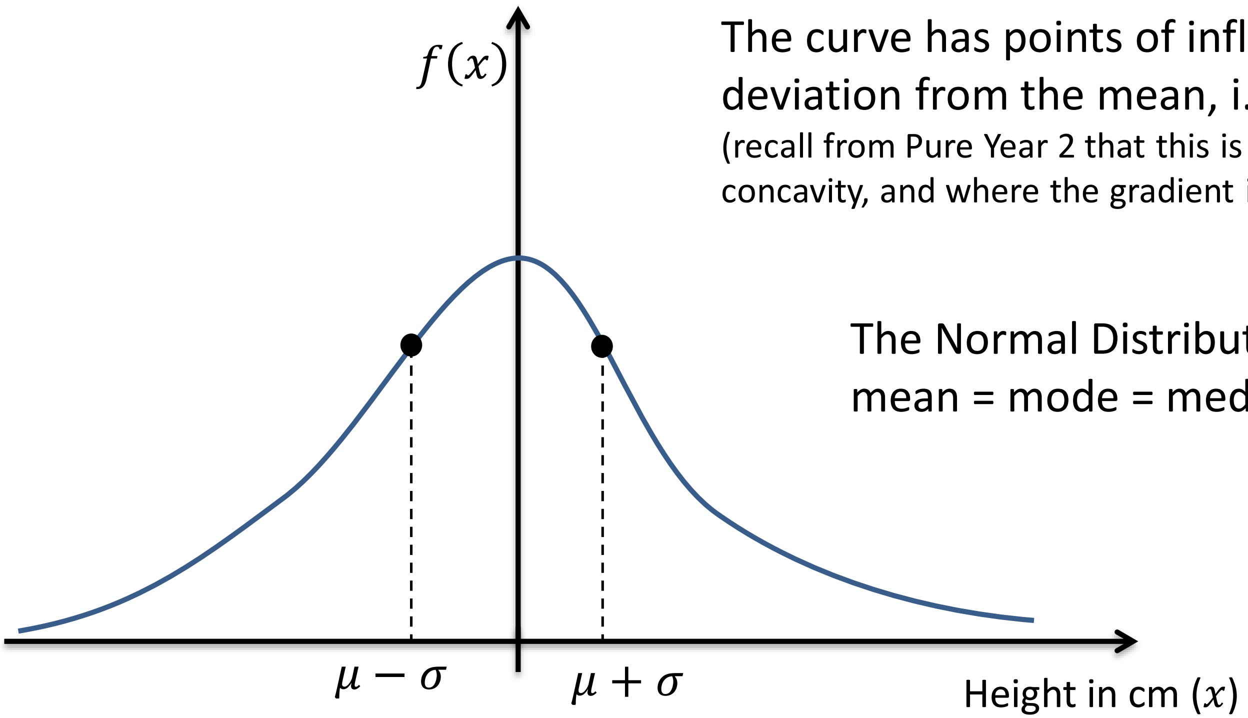
?

Q4 Would we ever want to find $P(X = 200)$ say?

?

Side Notes: You might therefore wonder what the y-axis actually is. It is **probability density**, i.e. "the probability per unit cm". This is analogous to frequency density with histograms, where the y-value is frequency density area under the graph gives frequency. We use $f(x)$ rather than $p(x)$, to indicate probability density.

Further Facts



The curve has points of inflection one standard deviation from the mean, i.e. $\mu \pm \sigma$
(recall from Pure Year 2 that this is where the curve changes concavity, and where the gradient is not changing)

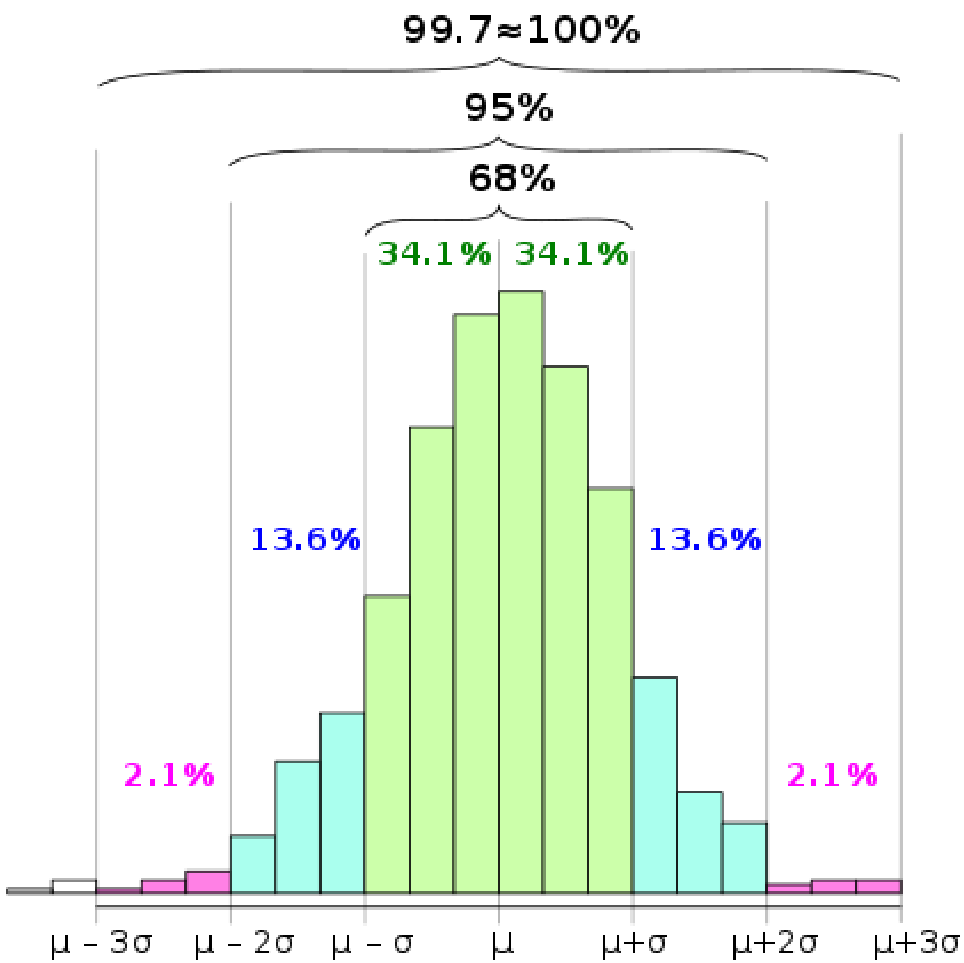
The Normal Distribution is symmetrical, i.e. mean = mode = median

Just For Your Interest™: The distribution, with a given mean μ and given standard deviation σ, that '**assumes the least**' (i.e. has the **maximum possible 'entropy'**) is… the Normal Distribution!

Difficult Proof: https://en.wikipedia.org/wiki/Differential_entropy#Maximization_in_the_normal_distribution

Extra Context: This is important in something called *Bayesian Statistics*. We often have to choose a suitable distribution for the 'prior' in the model (i.e. some 'hidden' variable). When making inferences based on observed data, we want to assume *as little as possible* about any hidden variable, so using a Normal distribution therefore is the most mathematically appropriate choice.

The 68-95-99.7 rule



The histogram above is for a quantity which is approximately normally distributed.
Source: Wikipedia

The 68-95-99.7 rule is a shorthand used to remember the percentage of data that is within 1, 2 and 3 standard deviations from the mean respectively.
You need to memorise this!

✏

$\approx$ 68% of data is within one standard deviation of the mean.
$\approx$ 95% of data is within two standard deviations of the mean.
$\approx$ 99.7% of data is within three standard deviations of the mean.

For practical purposes we consider all data to lie within $\mu \pm 5\sigma$

Only one in 1.7 million values fall outside $\mu \pm 5\sigma$. CERN used a "5 sigma level of significance" to ensure the data suggesting existence of the Higgs Boson wasn't by chance: this is a 1 in 3.5 million chance (if we consider just one tail).

Examples

[Textbook] The diameters of a rivet produced by a particular machine, X mm, is modelled as $X \sim N(8, 0.2^2)$. Find:

a) $P(X > 8)$

b) $P(7.8 < X < 8.2)$

Fro Tip: Draw a diagram!

a

?

b

?

Test Your Understanding

IQ ("Intelligence Quotient") for a given population is, by definition, distributed using $X \sim N(100, 15^2)$. Find:

a) $P(70 < X < 130)$

b) $P(X > 115)$

Fro Tip: Draw a diagram!

a

?

b

?

Exercise 3A

Pearson Stats/Mechanics Year 2
Pages 40-41

Getting normal values from your calculator

"Use of Technology" Monkey says:
If for some reason you haven't got a Classwiz by this point, for Kong's sake, buy one now. [Surreptitiously pockets banana handed to me by Casio]. The instructions below assume you have a Classwiz.

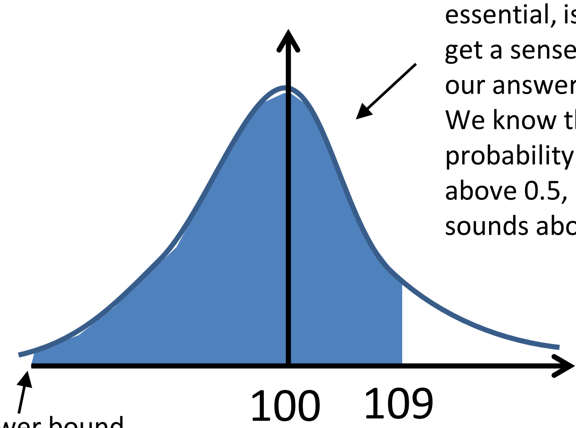
Just like a cumulative frequency graph gives the running total of the frequency up to a given value, a **cumulative distribution** gives the **running total of the probability** up to a given value.

IQ is distributed using $X \sim N(100, 15^2)$. Find
(a) $P(X < 109)$
(b) $P(X \geq 93)$
(c) $P(110 < X < 120)$
(d) $P(X < 80 \; or \; X > 106)$

a

1. Press MODE.
2. Choose DISTRIBUTION (option 7)
3. Choose Normal CD (i.e. "Cumulative Distribution")
4. Since the lower value is effectively $-\infty$, use any value at least 5σ below the mean (-100000 will do!). Press = after each value.
5. Put the upper value as 109.
6. Set $\sigma = 15$ and $\mu = 100$
7. You should obtain $P(X < 109) = 0.7257 \; (4dp)$

A diagram, while not essential, is useful to get a sense of whether our answer is correct. We know the probability must be above 0.5, so 0.7257 sounds about right.



No lower bound, so use -100000

Getting normal values from your calculator

IQ is distributed using $X \sim N(100, 15^2)$. Find

(a) $P(X < 109)$

(b) $P(X \geq 93)$

(c) $P(110 < X < 120)$

(d) $P(X < 80 \text{ or } X > 106)$

b

?

Fro Note: $\geq$ vs $>$ makes no difference as the distribution is continuous.

c

?

d

?

Test Your Understanding

The criteria for joining Mensa is an IQ of at least 131.
Assuming that IQ has the distribution $X \sim N(100, 15^2)$ for a population, determine:

a) What percentage of people are eligible to join Mensa.
b) If 30 adults are randomly chosen, the probability that at least 3 of them will be eligible to join. (Hint: Binomial distribution?)

a ?

b ?

Exercise 3B

Pearson Stats/Mechanics Year 2

Pages 40-41

Inverse Normal Distribution

We now know how to use a calculator to value of the variable to obtain a probability. But we might want to do the reverse: given a probability of being in a region, how do we find the value of the boundary?

[Textbook] $X \sim N(20, 3^2)$. Find, correct to two decimal places, the values of a such that:

a. $P(X < a) = 0.75$

b. $P(X > a) = 0.4$

c. $P(16 < X < a) = 0.3$

b

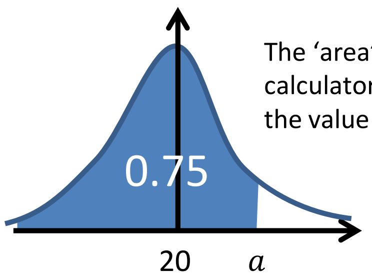
?

c

?

a

1. MODE → Distributions
2. Choose 'Inverse Normal'.
3. Put the area as 0.75 (this is the area up to the a value to determine). Put $\mu = 20$ and $\sigma = 3$.
4. You should get **22.0235**.



The 'area' requested by your calculator is the probability **up to** the value of interest (in this case a)

DRAW A SKETCH!

Further Example

If the IQ of a population is distributed using $X \sim N(100, 15^2)$.

a. Determine the IQ corresponding to the top 30% of the population.
b. Determine the interquartile range of IQs.

a

?

b

?

Test Your Understanding

$X \sim N(80, 7^2)$. Using your calculator,

a. determine the a such that $P(X > a) = 0.65$
b. determine the b such that $P(75 < X < b) = 0.4$
c. determine the c such that $P(c < X < 76) = 0.2$
d. determine the interquartile range of X.

a

?

c

?

b

?

d

?

Exercise 3C

Pearson Stats/Mechanics Year 2

Pages 46-47

Standard Normal Distribution

✎ Z is the number of standard deviations above the mean.

If again we use IQ distributed as $X \sim N(100, 15^2)$ then: (in your head!)

IQ	Z
100	?
130	?
85	?
165	?
62.5	?

This formula makes sense if you think about the definition above. For an IQ of 130: $Z = \frac{130-100}{15} = 2$ as expected.

✎ Z represents the coding:

$$Z = \frac{X - \mu}{\sigma}$$

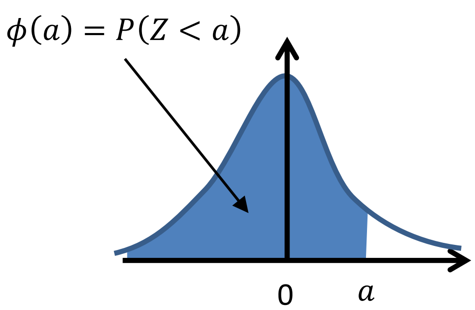
and $Z \sim N(0, 1^2)$. Z is known as a **standard** normal distribution.

The 0 and 1 of $Z \sim N(0, 1^2)$ are the result of the coding. If we've subtracted μ from each value the mean of the normal distribution is now 0. If we've divided all the values by σ the standard deviation is now $\frac{\sigma}{\sigma} = 1$

Standard Normal Distribution

The point of coding in this context is that all different possible normal distributions become a single unified distribution where we no longer have to worry about the mean and standard deviation. It means for example when we calculate $P(Z < 3)$, this will always give the same probability regardless of the original distribution.

It also means we can look up probabilities in a ***z*-table**:



✎ $\Phi(a) = P(Z < a)$ is the cumulative distribution for the standard normal distribution. The values of $\Phi(a)$ can be found in a z-table.

This is a traditional z-table in the old A Level syllabus (but also found elsewhere). You no longer get given this and are expected to use your calculator.

THE NORMAL DISTRIBUTION FUNCTION

The function tabulated below is $\Phi(z)$, defined as $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2}\, dt$.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.9772
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.02	0.9783
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.04	0.9793
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.06	0.9803
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.08	0.9812
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.10	0.9821
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.12	0.9830
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.14	0.9838
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.16	0.9846
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.18	0.9854
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.20	0.9861

This is from the new formula booklet. This is sometimes known as a 'reverse z-table', because you're looking up the z-value for a probability. Beware: p here it the probability of **exceeding** z rather than being up to z. Let's use it…

Percentage Points of The Normal Distribution

The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p; that is, $P(Z > z) = 1 - \Phi(z) = p$.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Examples

[Textbook] The random variable $X \sim N(50, 4^2)$. Write in terms of $\Phi(z)$ for some value of z.
(a) $P(X < 53)$ (b) $P(X \geq 55)$

a

?

b

?

[Textbook] The systolic blood pressure of an adult population, S mmHg, is modelled as a normal distribution with mean 127 and standard deviation 16. A medical research wants to study adults with blood pressures higher than the 95th percentile. Find the minimum blood pressure for an adult included in her study.

?

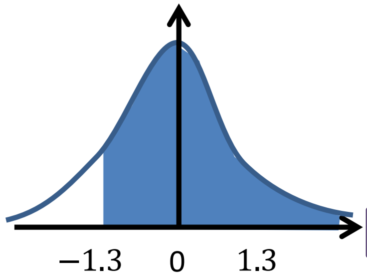
p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Further Examples

(a) Determine $P(Z > -1.3)$
(b) Determine $P(-2 < Z < 1)$
(c) Determine the a such that $P(Z > a) = 0.7$
(d) Determine the a such that $P(-a < Z < a) = 0.6$

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

a



?

b

?

Fro Tip: Either changing < to/from > or changing the sign (+ to/from -) has the effect of "1 −". However, if you change both, the "1 −"s cancel out!

$$P(a < Z < b) = P(Z < b) - P(Z < a)$$

c

?

d

?

Test Your Understanding

1 IQ is distributed with mean 100 and standard deviation 15. Using an appropriate table, determine the IQ corresponding to the
(a) top 10% of people.
(b) bottom 20% of people.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

a ?

b ?

2 If $X \sim N(100, 15^2)$, determine, in terms of Φ:
(a) $P(X > 115)$
(b) $P(77.5 < X < 112)$

a ?

b ?

3 Find the a such that:
(a) $P(-a < Z < a) = 0.2$
(b) $P(0 < Z < a) = 0.35$

a ?

b ?

Exercise 3D

Pearson Stats/Mechanics Year 2

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Missing μ and σ

In the last section, you may have thought, "what's the point of standardising to Z when I can just use the DISTRIBUTION mode on my calculator?"
Fair point, but both forward and reverse normal lookups on the calculator **required you to specify μ and σ.**

[Textbook] $X \sim N(\mu, 3^2)$. Given that $P(X > 20) = 0.2$, find the value of μ.

?

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

[Textbook] A machine makes metal sheets with width, X cm, modelled as a normal distribution such that $X \sim N(50, \sigma^2)$.
(a) Given that $P(X < 46) = 0.2119$, find the value of σ.
(b) Find the 90th percentile of the widths.

?

The method here is exactly the same as before:
1. Using a sketch, determine whether you're expecting a positive or negative z value.
2. Look up z value, using tables if you can (otherwise your calculator). Make negative if in bottom half.
3. Use $Z = \frac{X-\mu}{\sigma}$

When both are missing

If both μ and σ are missing, we end up with simultaneous equations which we must solve.

Edexcel S1 Jan 2011

The weight, Y grams, of soup put into a carton by machine B is normally distributed with mean μ grams and standard deviation σ grams.

(*c*) Given that $P(Y < 160) = 0.99$ and $P(Y > 152) = 0.90$, find the value of μ and the value of σ.

(6)

?

"Use of Technology" Monkey says:
Your Classwiz solves simultaneous equations. Look under the EQUATIONS mode.

Test Your Understanding

Edexcel S1 May 2013 (R)

The time taken to fly from London to Berlin has a normal distribution with mean 100 minutes and standard deviation d minutes.

Given that 15% of the flights from London to Berlin take longer than 115 minutes,

(b) find the value of the standard deviation d.

(4)

?

Edexcel S1 Jan 2002

5. The duration of the pregnancy of a certain breed of cow is normally distributed with mean μ days and standard deviation σ days. Only 2.5% of all pregnancies are shorter than 235 days and 15% are longer than 286 days.

(a) Show that $\mu - 235 = 1.96\sigma$.

(2)

(b) Obtain a second equation in μ and σ.

(3)

(c) Find the value of μ and the value of σ.

(4)

(d) Find the values between which the middle 68.3% of pregnancies lie.

(2)

(a) ?

(b) ?

(c) ?

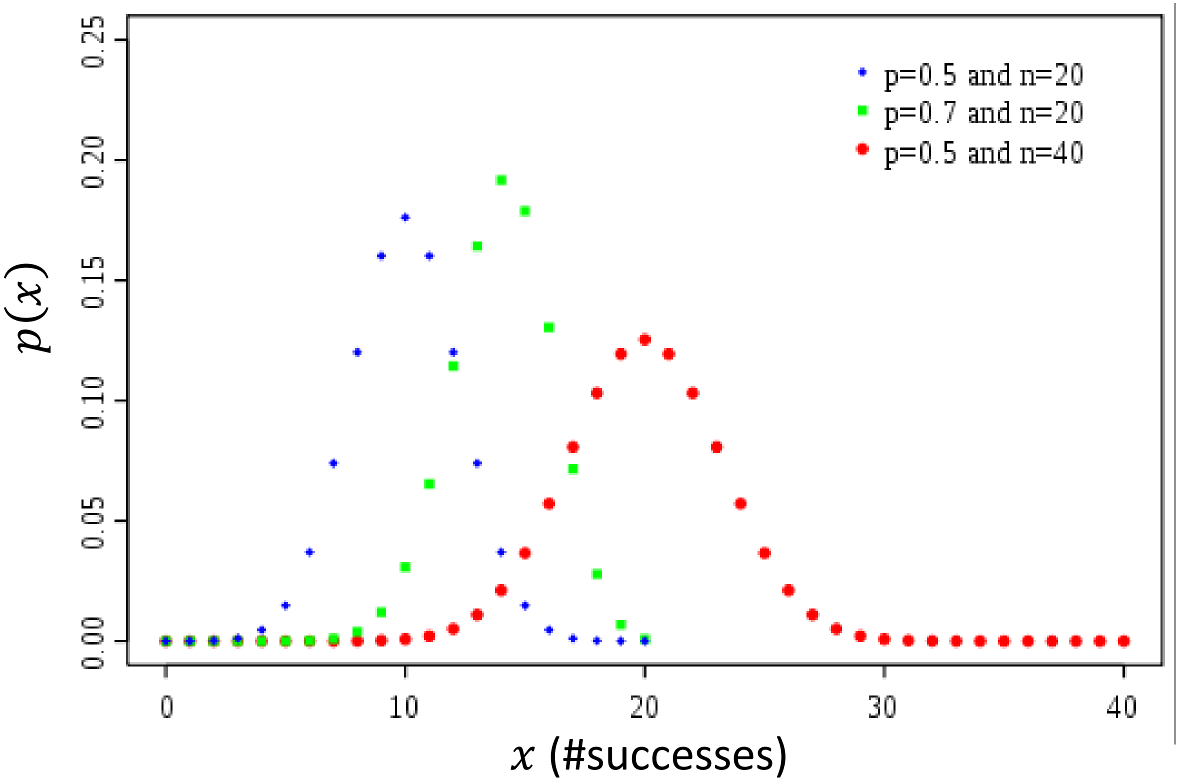
(d) ?

Exercise 3E

Pearson Stats/Mechanics Year 2

Pages 51-53

Approximating a Binomial Distribution



The graph shows the probability function for different Binomial Distributions. Which one resembles another distribution and what distribution does it resemble?

?

Approximating a Binomial Distribution

If we're going to use a normal distribution to approximate a Binomial distribution, it makes sense that we set the mean and standard deviation of the normal distribution to match that of the original binomial distribution:

$\mu =$?

$\sigma =$?

✎ If n is large and p close to 0.5, then the binomial distribution $X \sim B(n, p)$ can be approximated by the normal distribution $N(\mu, \sigma^2)$ where

$$\mu = np$$

$$\sigma = \sqrt{np(1-p)}$$

Quickfire Questions:

$X \sim B(10, 0.2) \quad \rightarrow \quad Y \sim N($?

$X \sim B(20, 0.5) \quad \rightarrow \quad Y \sim$?

$X \sim B(6, 0.3) \quad \rightarrow \quad Y \sim$?

We tend to use the letter Y to represent the normal distribution approximation of the distribution X.

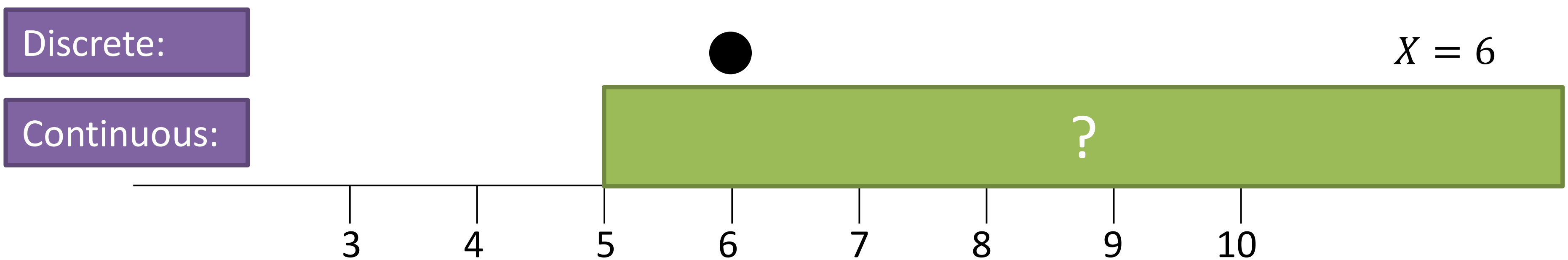
Why use a normal approximation?

- Tables for the binomial distribution only goes up to $n = 50$ and your calculator will reject large values of n.
- The formula for $P(X = x)$ makes use a factorials. Factorials of large numbers cannot be computed efficiently. Type in 65! for example; your calculator will hesitate! Now imagine how many factorials would be required if you wanted to find $P(X \leq 65)$. ☺

Continuity Corrections

One problem is that the outcomes of a binomial distribution (i.e. number of successes) are **discrete** whereas the Normal distribution is **continuous**.
We apply something called a **continuity correction** to approximate a discrete distribution using a continuous one.

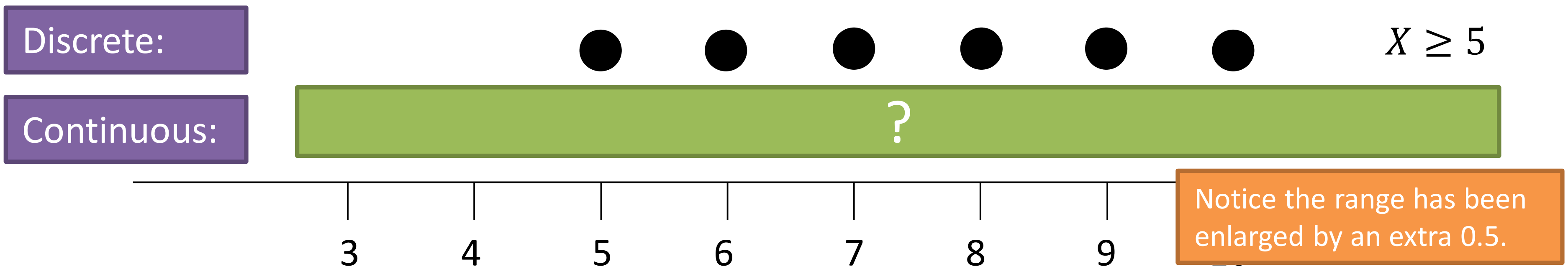
The random variable X represents the time to finish a race in hours. We're interested in knowing the probability Alice took 6 hours to the nearest hour. How would you represent this time on a number line given hours is discrete? And what about if hours was now considered to be continuous (as Y)?



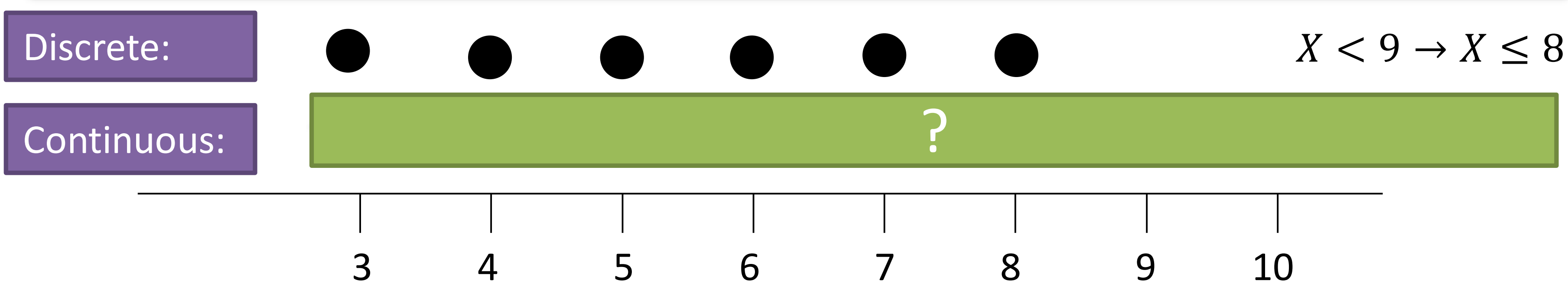
We can't just find $P(Y = 6)$ when Y is continuous, because the probability is effectively 0. But $P(5.5 < Y < 6.5)$ would seem a sensible interval to use because any time between 5.5 and 6.5 would have rounded to 6 hours were it discrete.

Continuity Corrections

If X is a discrete variable, and Y is its continuous equivalent, how would you represent $P(X \geq 5)$ for Y?



How would represent $P(X < 9)$ for Y?



✎ A continuity correction is approximating a discrete range using a continuous one.

1. If > or <, convert to ≥, ≤ first.
2. Enlarge the range by 0.5.

Examples

Discrete	Continuous
$P(X \leq 7)$	?
$P(X < 10)$	?
$P(X > 9)$	?
$P(1 \leq X \leq 10)$	?
$P(3 < X < 6)$	?
$P(3 \leq X < 6)$	?
$P(3 < X \leq 6)$	?
$P(X = 3)$	?

✎ A continuity correction is approximating a discrete range using a continuous one.

1. If > or <, convert to $\geq, \leq$ first.
2. Enlarge the range (at each end) by 0.5.

Full Example

[Textbook - Edited] For a particular type of flower bulbs, 55% will produce yellow flowers. A random sample of 80 bulbs is planted.

(a) Calculate the actual probability that there are exactly 50 flowers.

(b) Use a normal approximation to find a estimate that there are exactly 50 flowers.

(c) Hence determine the percentage error of the normal approximation for 50 flowers.

a

?

b

?

c

?

Test Your Understanding

Edexcel S2 Jan 2004 Q3

The discrete random variable X is distributed $B(n, p)$.

(*a*) Write down the value of p that will give the most accurate estimate when approximating the binomial distribution by a normal distribution.
(1)

(*b*) Give a reason to support your value. **(1)**

(*c*) Given that $n = 200$ and $p = 0.48$, find $P(90 \leq X < 105)$. **(7)**

(a) ?

(b) ?

(c) ?

Exercise 3F

Pearson Stats/Mechanics Year 2

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Hypothesis Testing on the Sample Mean

0 1 2 3 4 5 6 7 8 9

Imagine we have 10 children, one of each age between 0 and 9.
This is our population. There is a **known population mean** of $\mu = 4.5$

					$\bar{x}$
Sample 1:	1	3	7	8	4.75
Sample 2:	6	2	0	9	4.25

…

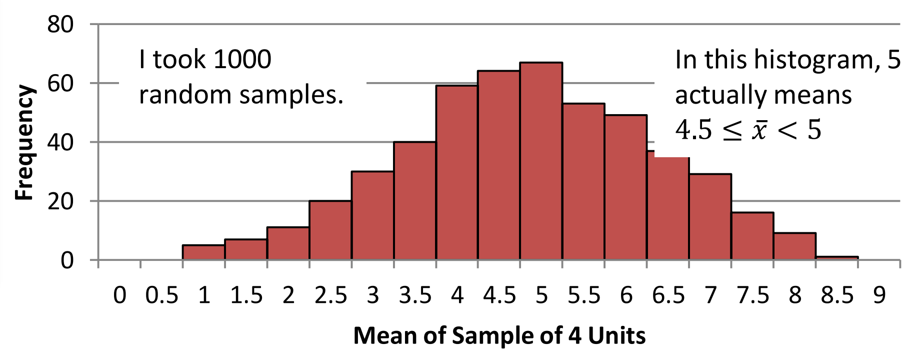
Suppose we took a sample of 4 children.

Sample mean $\bar{x}$	Tally
4.00	
4.25	|
4.50	
4.75	|
5.00	

The mean of this sample is $\bar{x} = 4.75$. This sample mean $\bar{x}$ is close the true population mean μ, but is naturally going to vary as we consider different samples.

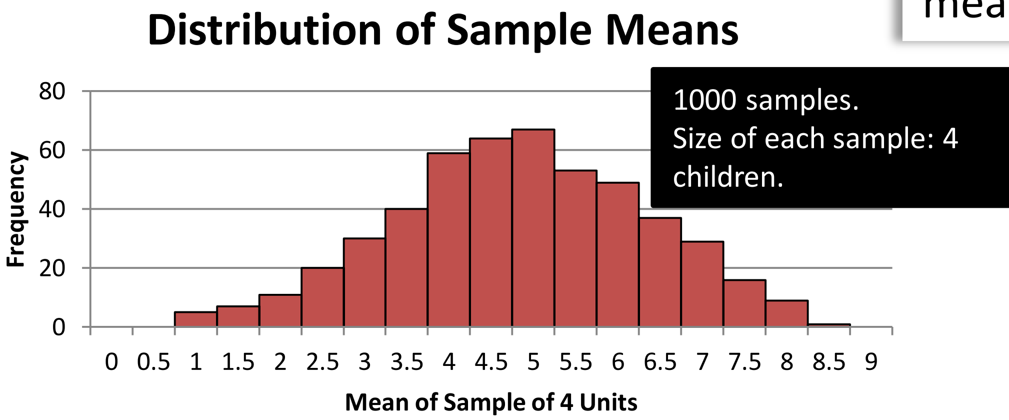
For a different sample of 4, we might obtain a different sample mean. What would happen if we took lots of different samples of 4, and found the mean $\bar{x}$ of each? How would these means be distributed?

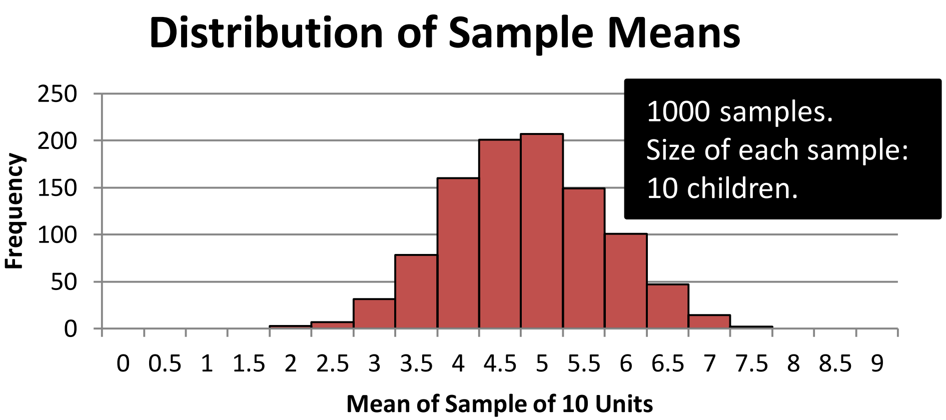
Distribution of Sample Means $\bar{X}$



Hypothesis Testing on the Sample Mean

$\bar{X}$ is our distribution across different sample means as we consider different samples.





✐ For a random sample of size n taken from a random variable X, the sample mean $\bar{X}$ is normally distributed with $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Question 1: What type of distribution is $\bar{X}$?

?

Question 2: On average, what sample mean do we see? (i.e. the mean of the means!)

?

Question 3: Is the variance of $\bar{X}$ (i.e. how spread out the sample means are) the same as that of the variance of the population of children?

?

Examples

[Textbook] A certain company sells fruit juice in cartons. The amount of juice in a carton has a normal distribution with a standard deviation of 3ml.
The company claims that the mean amount of juice per carton, μ, is 60ml. A trading inspector has received complaints that the company is overstating the mean amount of juice per carton and wishes to investigate this complaint. The trading inspector takes a random sample of 16 cartons and finds that the mean amount of juice per carton is 59.1ml.

Using a 5% level of significance, and stating your hypotheses clearly, test whether or not there is evidence to uphold this complaint.

H_0: ?
H_1: ?
Let X be ?
Assuming H_0, ?
$\therefore \bar{X} \sim$?

?

As always, state the distribution X and its distribution under H_0.

Use X to work out the distribution of the sample means, $\bar{X}$.

Fro Note: Don't confuse X and $\bar{X}$. The X is the distribution over amounts of drink in each individual carton. $\bar{X}$ is the distribution over **sample means**, i.e. the possible sample means we see as we take samples of 16 cartons. X might not be normally distributed, but $\bar{X}$ will be.

Finding the critical region

[Textbook] A machine products bolts of diameter D where D has a normal distribution with mean 0.580 cm and standard deviation 0.015 cm. The machine is serviced and after the service a random sample of 50 bolts from the next production run is taken to see if the mean diameter of the bolts has changed from 0.580 cm. The distribution of the diameters of bolts after the service is still normal with a standard deviation of 0.015 cm.

(a) Find, at the 1% level, the critical region for this test, stating your hypotheses clearly.
The mean diameter of the sample of 50 bolts is calculated to be 0.587 cm.
(b) Comment on this observation in light of the critical region.

a H_0: ?

Assuming H_0, ?

$\bar{D} \sim$?

$z = \pm$?

?

b ?

This is a two-tailed test so want the z value corresponding to the top 0.5% and bottom 0.5% respectively. Use table below or your calculator.

Use the usual formula $Z = \frac{X-\mu}{\sigma}$, except this time we use the standard deviation of $\bar{X}$: $\sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

Test Your Understanding

Edexcel S3 June 2011 Q7a

Roastie's Coffee is sold in packets with a stated weight of 250 g. A supermarket manager claims that the mean weight of the packets is less than the stated weight. She weighs a random sample of 90 packets from their stock and finds that their weights have a mean of 248 g and a standard deviation of 5.4 g.

(*a*) Using a 5% level of significance, test whether or not the manager's claim is justified. State your hypotheses clearly.

(5)

(a)

?

Exercise 3G

Pearson Stats/Mechanics Year 2

Pages 58-60

Conditional Probabilities

This is not in the textbook. But given the recent Chapter 2 on Conditional Probabilities and the fact that the type of question below occurred frequently in S1 papers, it seems worthwhile to cover!

Edexcel S1 May 2014(R) Q4

The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

(c) find $P(X > \mu + 15 \mid X > \mu - 15)$. **(3)**

?

Test Your Understanding

Edexcel S1 Jan 2013 Q4a,c

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(*a*) Find $P(L > 127)$.

(3)

Alice is about to go on a 6 hour journey. Given that it is 127 hours since Alice last charged her phone,

(*c*) find the probability that her phone will not need charging before her journey is completed.

(4)

(a) ?

(c) ?

One Last Toughie…

IQ is distributed as $X \sim N(100, 15^2)$.
A person is considered to be 'pretty smart' if their IQ is at least 130.
Determine the median IQ of 'pretty smart' people.

?